



# *Dynamic Response of Sandwich Panels with Flexible Cores and Elastic Foundation Subjected to Low-Velocity Impact*

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## **ABSTRACT**

In this paper, for first time dynamic response of composite sandwich panel with a flexible core on elastic foundation under low-velocity impact load is studied. The governing equations of sandwich panel are obtained using the Hamilton principle and energy method. Free vibration analysis of panel is based on the improved higher-order sandwich panel theory (IHSAPT) by considering elastic foundation effects in equations. The formulation uses the first shear deformation theory (FSDT) for face sheets and Frostig's second model for core. The equations system is solved for plate on elastic foundation using two parameters Pasternak model. For dynamic analysis of composite sandwich panel under low-velocity impact load, the sandwich panel is modeled as a two-degrees-of-freedom dynamic system with equivalent masses and springs using linearized Hertz contact law from Choi method. In this model for impact solution, fundamental natural frequency has been used directly for calculation of the equivalent stiffness of panel. Effect of variation of various vertical and transverse shear modulus of elastic foundation on natural frequencies of free vibration, contact force history and transverse displacement of panel in low-velocity impact analysis is discussed and is compared with the available results in literature.

## **KEYWORDS**

Sandwich Panel, Flexible Core, Low-Velocity Impact, Elastic Foundation, Vibration, High- Order Theory.

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**1- INTRODUCTION**

Frostig and Thomsen [1] present two models of high order theory for vibration of sandwich panels. Choi and Lim could achieve the linear Hertzian impact model [2]. Malekzadeh and Khalili implemented new mass-spring-damper-dashpot model with three degrees of freedom for calculating of contact force history on sandwich panel [3].

Malekzadeh et al. have proposed the modified high order theory for sandwich panels with modified Frostig's theory[3]. Chien and Chen[4] presented the nonlinear vibration of laminated plates on an elastic foundation. Yas and Sobhani [5] studied free vibration analysis of continuous grading fiber reinforced plates on elastic foundation. They included effects of elastic foundation in higher order differential equations of the system. Malekzadeh [6] studied free vibration analysis of fiber reinforced composite thick plates on elastic foundation. Anderson [7] presented a single degree-of-freedom model for large mass impact on composite sandwich laminates. Stiffness parameters of the models were derived from the results of three-dimensional quasi-static contact analyses of a rigid sphere indenting on multi-layer sandwich laminate. The energy balance and mass-spring model are some of main equivalent models. They are a reasonable approach in special cases for Impact dynamic assessment. In this paper, for first time, dynamic response of composite sandwich panel with a flexible core on elastic foundation under low-velocity impact is studied. For simulation of low-velocity impact, the mass-spring model has been used with two degrees of freedom [2].

**2- METODOLOGY**

The differential equations of sandwich panels with flexible cores could be achieved by equation(1) [3]:

$$\delta \int_{t_1}^{t_2} (U + V - T + U_f) dt = 0 \tag{1}$$

Strain energy for sandwich panel on elastic foundation could be represented by equation (2) [4]:

$$U_f = \int_s f_e w_b ds \tag{2}$$

$s$  is the area of plate on elastic foundation and  $f_e$  is the reaction force of elastic foundation per unit area of surface which with the two-parameter Pasternak's model is quoted by equation (3)[4]:

$$f_e = k_w w_b - k_g \frac{\partial^2 w_b}{\partial x^2} - k_g \frac{\partial^2 w_b}{\partial y^2} \tag{3}$$

For face sheets the first shear deformation theory (FSDT) has been used. Considering the represented equation in second Frostig's model [1] and the achieved results by Malekzadeh et al.[3], the displacements in different directions in the core are assumed as polynomial descriptions with unknown coefficients. The number of equations is 15. In order to solve the eigen value problem, Fourier series functions have been used for simply supported boundary conditions of the panel. Natural frequencies of panel are resulted from eigenvalue equations(4):

$$([K] - [M]\omega^2)\{X_0^*\} = \{0\} \tag{4}$$

Regarding mass-spring system with two degrees of freedom[3], the equations of motion would be resulted as below:

$$\begin{cases} m_1 \ddot{\Delta}_1 + K_c^*(\Delta_1 - \Delta_2) = 0 \\ M_{eff}^p \ddot{\Delta}_2 + K_c^*(\Delta_2 - \Delta_1) + K_g \Delta_2 = 0 \end{cases} \tag{5}$$

By solving these equations, the general results of equations (5) are quoted by equation (6):

$$\begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix} = C_1 \varphi^{-(1)} \sin(\omega_{n1} t + \psi_1) + C_2 \varphi^{-(2)} \sin(\omega_{n2} t + \psi_2) \tag{6}$$

By identifying the unknowns of equation (6), the contact force function would be obtained as:

$$F(t) = K_c^*(\Delta_1 - \Delta_2) \tag{7}$$

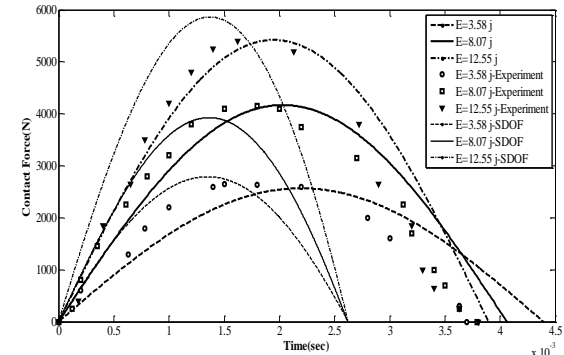
**3- RESULTS**

The non-dimensional fundamental natural frequencies of the panel with different values of Winkler (normal) and Pasternak (shear) foundations have been calculated by the present method and compared with the results of references [5] and [6]. These results have been shown in Table (1).

**Table (1)**

$b/h$	$K_w$	$K_g$	$\Omega_{11}$	$\Omega_{11}$
			[5],[6]	present
2	100	100	2.1	2.0
2	100	10	1.7	1.8
2	100	1	1.5	1.5
2	0	0	-	1.3
2	0	10	1.6	1.6
2	10	10	1.6	1.7
2	100	10	1.7	1.8
5	0	0	-	1.7
5	0	10	2.2	2.2
5	10	10	2.2	2.2
5	100	10	2.4	2.4
5	1000	10	3.7	3.7

For low velocity impact analysis of sandwich panel with elastic foundation, a square sandwich panel with simple supported boundary conditions with ply sequence (O<sub>2</sub>/90<sub>2</sub>/O<sub>2</sub> /core/ O<sub>2</sub>/90<sub>2</sub>/O<sub>2</sub>) was considered. The geometrical and mechanical properties of sandwich plate are available in reference [7].



**Diagram (1)**

In order to verify the present method, the obtained results were compared with the single degree of freedom (SDOF) nonlinear model [7]. Effects of impactor energy on contact force histories were shown in Diagram (1).

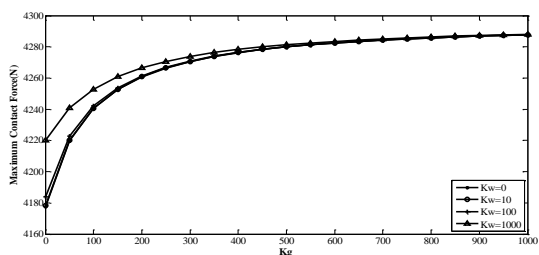


Diagram (2)

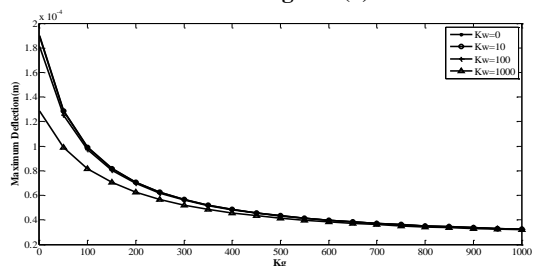


Diagram (3)

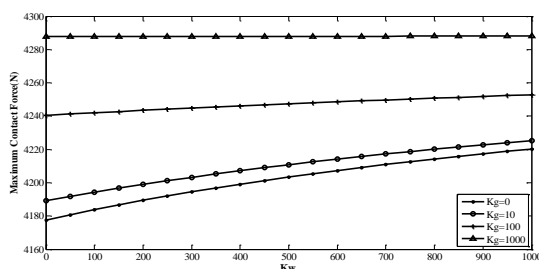


Diagram (4)

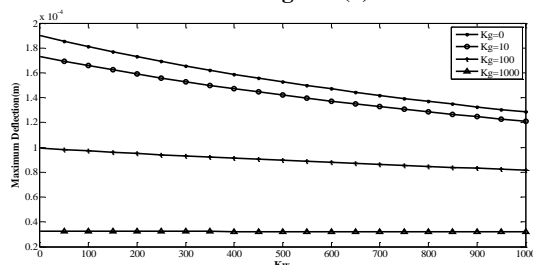


Diagram (5)

Regarding to the Table (1), the amount of natural frequency has small variations with increasing the normal module. But it has significant variations with increasing the shear module. According to the Diagrams (2-5), the fundamental natural frequency and impact force would be increased with increasing the amount of normal and shear modules, but the amount of deflection would be decreased with increasing them. Also the variations of maximum contact force and deflection have linear variation with respect to increase of normal module, but they have a nonlinear variation with respect to increase of shear module. It is obvious that the shear module has more effect on the contact force and deflection variations.

#### 4- CONCLUSION

In this paper, using an improved fully dynamic higher order sandwich panel theory, dynamic response of a sandwich panel with a functionally graded core under low velocity impact was studied. By considering the elastic foundation, the second Frostig's model was enhanced by introducing the first order shear deformation theory (FSDT) in the face-sheets and incorporating inertia forces and internal in-plane stresses of the functionally graded core into governing equations of motion. The effects of elastic foundation parameters on natural frequencies and low velocity response of sandwich panel were considered and discussed. As seen from the results, the present model was in excellent agreement with alternative solutions and results of the experimental test for single impact. It is obvious that the shear module has more effect on the contact force and deflection histories.

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