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Robust Adaptive Backstepping Control of a Nonlinear System with Uncertainty, Disturbance and Unknown Time Delay

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ABSTRACT

In this paper, robust adaptive control is presented for a class of nonlinear systems in strict feedback form with uncertain time delay. It is assumed that time delay is not known, thus terms having delays must not appear in adaptation and control laws. By using the Lyapunov-Krasovskii functional, terms having time delay are deleted from adaptation and control laws. The adaptive backstepping method is used to design a controller and it is shown that this controller guarantees global uniform asymptotic stability of the system. A controller is robust against uncertain time delay and bounded disturbances, which enter the system. Two simulation results are provided to show the effectiveness of the proposed approach.

KEYWORDS

Robust Adaptive Control, Unknown Time Delay, Bounded Disturbance, Backstepping, Lyapunov-Krasovskii Functional.

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1– INTRODUCTION

In recent years, the problem of robust control of nonlinear uncertain systems has attracted many researchers [1]. Several adaptive approaches for nonlinear systems with a triangular structure have been proposed in [2,3]. Robust adaptive control has been studied for a certain class of nonlinear systems whose uncertainties are not only from parametric ones but also from unknown functions in [4].

Time delays appear in many physical systems, such as rolling mill systems, biological systems, smart materials, etc. Time delay may cause instability of the physical systems and disrupt controller performance. Therefore, the stability problem of time delay systems has attracted the attention of many researchers in recent years.

One major difficulty that lies in time delayed systems is that the delays are not usually perfectly known. Few attempts have been made towards nonlinear systems with unknown time delay.

In practical applications, there is a possibility that disturbance signals apply to the system, which may lead to system instability. Disturbance signals, which may be applied to physical systems, are usually bounded and we can consider upper limits for them.

In the previous works, systems with three simultaneous difficulties 1- unknown time delay, 2- parameter uncertainty and 3- disturbance signals have not been considered. Therefore, we consider a nonlinear system in parametric strict feedback form with uncertain parameters and unknown time delay. Moreover, it is assumed that the system is affected by bounded disturbance signals. We employ robust adaptive backstepping control for this system and design state feedback control in order to stabilize it under unknown time delay and disturbance signals. The unknown time delay is compensated for by using appropriate Lyapunov-Krasovskii functional. We show that this controller is robust against uncertain time delay and bounded disturbance signals.

2– METHODOLOGY

Consider a class of single input single output (SISO) nonlinear time delay system:

$$\begin{cases} \dot{x}_1 = x_2 + \boldsymbol{\theta}^T \mathbf{f}_1(x_1) + \boldsymbol{\psi}^T \mathbf{h}_1(x_{1,\tau_1}) + d_1 \\ \dot{x}_2 = x_3 + \boldsymbol{\theta}^T \mathbf{f}_2(\mathbf{x}) + \boldsymbol{\psi}^T \mathbf{h}_2(\mathbf{x}_{\tau_2}) + d_2 \\ \vdots \\ \dot{x}_n = u + \boldsymbol{\theta}^T \mathbf{f}_n(\mathbf{x}) + \boldsymbol{\psi}^T \mathbf{h}_n(\mathbf{x}_{\tau_n}) + d_n \end{cases} \quad (1)$$

$$y = x_1$$

Where $\mathbf{x}, u \in \mathbb{R}, y \in \mathbb{R}$ are state variables, system input and output, respectively. $\mathbf{f}_i(\cdot)$ and $\mathbf{h}_i(\cdot)$ are known smooth functions and τ_i are uncertain time delays of the states, $i = 1, \cdots, n$. $\boldsymbol{\theta}, \boldsymbol{\psi}$ are vectors of uncertain parameters and d_i are bounded disturbances and $\mathbf{x}_{\tau k} = \mathbf{x}(t - \tau_k)$.

By using the adaptive backstepping approach [6], we can find the appropriate Lyapunov function as (2).

$$V_n = \frac{1}{2}\sum_{i=1}^{n} z_i^2 + \frac{1}{2}\sum_{i=1}^{n} \tilde{\boldsymbol{\theta}}_i^T \boldsymbol{\Gamma}_{i,\theta}^{-1} \tilde{\boldsymbol{\theta}}_i + \frac{1}{2}\sum_{i=1}^{n} \Gamma_{i,\beta}^{-1} \tilde{\beta}_i^2 + \frac{1}{2}\sum_{i=1}^{n} (n-i+1)\int_{t-\tau_i}^{t} \mathbf{h}_i^T(s)\mathbf{h}_i(s)ds \quad (2)$$

By taking the time derivative of (2) and doing some calculations and simplifications for the whole system (1), adaptation and control laws are derived as follows:

$$\dot{\hat{\boldsymbol{\theta}}}_n = \boldsymbol{\Gamma}_{n,\theta}(\mathbf{f}_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j}\mathbf{f}_j) z_n, \quad \dot{\hat{\beta}}_n = \frac{1}{2}\Gamma_{n,\beta}[1 + \sum_{j=1}^{n-1}(\frac{\partial \alpha_{n-1}}{\partial x_j})^2] z_n^2 \quad (3)$$

$$u = \begin{cases} -kz_n - z_{n-1} - \mathbf{f}_n^T \hat{\boldsymbol{\theta}}_n - \frac{1}{2} z_n [1 + \sum_{i=1}^{n-1}(\frac{\partial \alpha_i}{\partial x_i})^2]\hat{\beta}_n \\ -\frac{1}{2z_n}\sum_{i=1}^{n} \mathbf{h}_i^T(\mathbf{x})\mathbf{h}_i(\mathbf{x}) - sign(z_n)[D_n + \sum_{i=1}^{n-1}(\frac{\partial \alpha_i}{\partial x_i})D_i] \\ +\sum_{i=1}^{n-1}\frac{\partial \alpha_{n-1}}{\partial x_i}(x_{i+1} + \mathbf{f}_i^T \hat{\theta}_n) + \sum_{i=1}^{n-1}\frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_i}\boldsymbol{\Gamma}_{i,\theta}(\mathbf{f}_i - \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_j}\mathbf{f}_j) z_i \\ +\frac{1}{2}\sum_{i=1}^{n-1}\frac{\partial \alpha_{n-1}}{\partial \hat{\beta}_i}\Gamma_{i,\beta}[1 + \sum_{j=1}^{i-1}(\frac{\partial \alpha_{n-1}}{\partial x_j})^2] z_i^2 & , z_n \neq 0 \\ 0, & z_n = 0 \end{cases}$$

(4)

3– RESULTS AND DISSCUSION

To show the effectiveness of the proposed approach we perform two simulation studies and compare them with other controllers. For the first example, consider a nonlinear time delayed system:

$$\begin{cases} \dot{x}_1 = x_2 + f_1\theta + \psi h_{1,\tau_1} + d_1 \\ \dot{x}_2 = gu + f_2\theta + \psi h_{2,\tau_2} + d_2 \end{cases}, \quad y = x_1 \quad (5)$$

In which

$$f_1 = x_1^2,\ h_1 = x_{1\tau}^3,\ d_1 = 0, D_1 = 0, f_2 = x_2^2 \sin(x_2),$$
$$h_2 = x_{2\tau}\sin(x_{1\tau}), d_2 = 0.3\sin(t) - 0.4\cos(t),\ D_2 = 0.51$$
$$\tau_1 = 2,\ \tau_2 = 1,\ \theta = 2,\ \psi = 2$$

According to (4), the control signal is in the following form:

$$\boldsymbol{\xi} = [\frac{\theta}{g}, \frac{\psi^2}{g}, \frac{1}{g}], \quad \dot{\hat{\theta}}_1 = \gamma_1 z_1 f_1, \dot{\hat{\beta}}_1 = \frac{1}{2}\gamma_2 z_1^2$$

$$\mathbf{F}_\xi = [f_2 - \frac{\partial \alpha_1}{\partial z_1} f_1; \frac{1}{2} z_2; \frac{1}{2z_2}(h_1^2 + h_2^2) + sign(z_2)D_2$$

$$-\frac{\partial\alpha_1}{\partial z_1}x_2 + sign(z_2)\left|\frac{\partial\alpha_1}{\partial x_1}\right|D_1 - \frac{\partial\alpha_1}{\partial\hat{\theta}_1}\gamma_1 z_1 f_1 - \frac{\partial\alpha_1}{\partial\hat{\beta}_1}\frac{1}{2}\gamma_1 z_1^2]$$

$$u = \begin{cases} -z_1 - k_1 z_2 - \hat{\xi}^T\,\mathbf{F}_\xi, & if\ z_2 \neq 0 \\ 0, & if\ z_2 = 0 \end{cases} \qquad (6)$$

Figure (1) shows the behavior of output x_1 of (5) by applying control input (6) and PID controller. It is obvious that controller (6) has better performance than the PID controller.

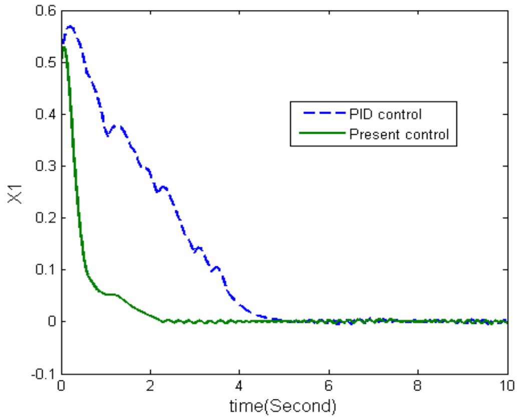

FIGURE (1) - Stabilization problem for system (5)

For the second example, consider the following nonlinear time delayed system:

$$\begin{cases} \dot{x}_1 = x_2 + f_{1,k}\theta + \psi h_{1,\tau_1} + d_1 \\ \dot{x}_1 = u + f_{2,k}\theta + \psi h_{2,\tau_2} + d_2 \end{cases}, \quad y = x_1 \qquad (7)$$

In which

$$f_1 = x_1^2,\ h_1 = x_{1\tau}^3,\ d_1 = 0, D_1 = 0, f_2 = x_2^2\sin(x_2),$$
$$h_2 = x_{2\tau}\sin(x_{1\tau}), d_2 = 0.3\sin(t)-0.4\cos(t),\ D_2 = 0.51$$
$$\tau_1 = 2,\ \tau_2 = 1,\ \theta=2,\ \psi = 2$$

According to (4), the control signal is obtained as:

$$u = \begin{cases} -z_1 - k_2 z_2 - \hat{\theta}_2 f_2 - \frac{1}{2}z_2\hat{\beta}_2 - sign(z_2)D_2 \\ +\frac{\partial\alpha_1}{\partial x_1}(x_2 + \hat{\theta}_2 f_1) - \frac{1}{2}(\frac{\partial\alpha_1}{\partial x_1})^2 z_1\hat{\beta}_1^2 \\ -\frac{1}{2z_2}(h_1^2 + h_2^2) + \frac{\partial\alpha_1}{\partial\hat{\theta}_1}z_1\gamma_1 f_1 \\ +\frac{\partial\alpha_1}{\partial\hat{\beta}_1}\frac{1}{2}\gamma_2 z_1^2 + \frac{\partial\alpha_1}{\partial t} \quad ; \quad z_2 \neq 0 \\ 0, \qquad\qquad z_2 = 0 \end{cases} \qquad (8)$$

Figure (2) shows a tracking problem for system (7). According to this figure, controller (8) has better performance than sliding mode control.

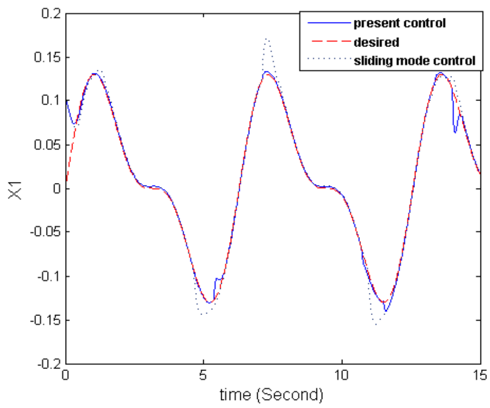

FIGURE (2) - Tracking problem for system (7)

4– CONCLUSIONS

A nonlinear system in strict feedback form has been studied. There were different complexities in the proposed system such as: parameter uncertainties, unknown time delays and bounded disturbances. By using backstepping design, a robust adaptive control has been designed to guarantee global uniform asymptotic stability of the closed loop system. The simulation study has been provided to show the effectiveness of the proposed approach.

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