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Numerical and Experimental Analysis of Free Vibration of Post-buckled Beam

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ABSTRACT

Vibration analysis of post-buckled beam is investigated in this study. Governing nonlinear equations of motion for the post-buckled state are derived by neglecting terms containing the damping effects, shear deformation and rotary inertia. The beam is assumed to have some geometrical imperfections, which are represented in terms of the first modal shape of the intact beam with different amplitudes. Considering the small amplitude vibrations around the post-buckled equilibrium configuration, the solution consists of static and dynamic parts, both leading to nonlinear differential equations. The differential quadrature method has been used to solve the problem. First, it is applied to the equilibrium equations, leading to a nonlinear algebraic system of equations that is solved utilizing an arc length strategy. Next, the differential quadrature is applied to the linearized dynamic differential equations of motion and their corresponding boundary and continuity conditions. Upon solution of the resulting eigenvalue problem, the natural frequencies and mode shapes of the beam are extracted. The investigation includes several numerical as well as experimental case studies on the post-buckled simply supported and clamped-clamped beams. The results show that the applied compressive load as well as the geometric imperfection largely affect the modal shapes and natural frequencies of the beam. Moreover, the study demonstrates the excellent accuracy and efficiency that can be obtained by applying the differential quadrature method to treat vibration of the post-buckled beams.

KEYWORDS:

Beam vibration, Post buckling, Differential quadrature method, Experimental modal analysis

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1- Introduction

The problem of vibration of an elastic beam around its post-buckled state has gained much attention recently. Nayfeh et al. [1] analyzed the vibrational mode shape of buckled beams using analytical and experimental methods. They used the static buckling mode shape and found an exact solution for natural frequency as well as vibrational mode shape of the beam. Neukirch et al. [2] studied the small amplitude in-plane vibrations of an elastic clamped-clamped rod around its post-buckled state using extensible and inextensible models, analytically and numerically. They found that while for some modes there are no qualitative changes in the mode frequencies, other frequencies experience rapid variations after the buckling threshold. This study investigates the vibrational behavior of a post-buckled elastic beam both numerically and experimentally.

2- Methodology

Consider an elastic beam of length l, height h and width b subjected to an axial compressive force p at $x=l$. Neglecting the damping effects, shear deformation and rotary inertia, governing differential equations of motion for the beams' post-buckled state can be presented by:

$$
\begin{aligned}
&\frac{\partial w}{\partial s} = \left(1+\frac{n}{EA}\right)\sin\theta - \sin\theta_0 \\
&\frac{\partial u}{\partial s} = \left(1+\frac{n}{EA}\right)\cos\theta - \cos\theta_0 \\
&EI\left[\frac{\partial \theta}{\partial s} - \frac{\partial \theta_0}{\partial s}\right] - m = 0 \\
&\frac{\partial m}{\partial s} + q = 0 \\
&\frac{\partial}{\partial s}\left(n\sin\theta\right) + \frac{\partial}{\partial s}\left(q\cos\theta\right) = \rho\ddot{w} \\
&\frac{\partial}{\partial s}\left(n\cos\theta\right) - \frac{\partial}{\partial s}\left(q\sin\theta\right) = \rho\ddot{u}
\end{aligned} \tag{1}
$$

where s is the arc length of the deflection curve, u and w are the displacements along the x and y axes, respectively. In addition, θ is the rotation with respect to the x axis, θ_0 is the initial rotation due to the geometric imperfection, and n, q and m are the axial and shear forces, and bending moment, respectively. The first two equations represent the strain components of the element, the third equation corresponds to the constitutive equation, and the last three represent the governing differential equations of the motion. A is the cross section and I is the second moment of inertia.

Considering the small amplitude vibrations around the post-buckled equilibrium configuration, the solution can be written as the sum of the equilibrium and harmonic parts. In order to solve the vibration of the post-buckled beams, first the system of Eq. (1) are solved statically to determine the equilibrium shape. Next, the small vibrations around the post-buckled equilibrium are considered. Removing the nonlinear terms, the linear dynamic equations of motion can be obtained.

The differential quadrature (DQ) method is utilized to solve both the nonlinear post-buckled equilibrium equations and linear dynamic equations of motion. The method states that the derivative of a function with respect to a space variable can be approximated by a weighted linear combination of function values at some intermediate points in the domain of that variable [3]. Application of the DQ method to the static post-buckled equations and their corresponding boundary conditions results in a system of nonlinear algebraic equations, which can be solved using an arc-length strategy.

Determining the post-buckled equilibrium state of the beam, next their small vibration around this state is considered. Discretizing the system of linear dynamic equations of motion and their corresponding boundary conditions result in a system of linear eigenvalue equations. The solution of this eigenvalue problem by a standard eigensolver provides the natural frequencies and corresponding modal shapes of the post-buckled beam.

3- Results and Discussion

To verify the effectiveness of the presented approach, an experimental study is carried out on clamped-clamped and simply supported beams made of polyvinylchloride (PVC). The beam considered here is 775 mm long, 20 mm width and 10 mm height. Modulus of elasticity, Poisson's ratio, and density of the PVC beam are 3.7 Gpa, 0.4 and 1400 kg/m3, respectively. Different fixtures are used to represent the simply supported and clamped boundary conditions (see Fig. 1). Then, the load is applied to the beam to produce the state of postbuckling with certain amounts of end shortening. A data acquisition system is utilized to measure the lowest three natural frequencies of the intact beams.

The natural frequencies obtained by the proposed method as well as the FEM results are presented in Table 1. The commercial ANSYS finite element software was used to model the problem. It used 500 BEAM189 elements to model the beam.

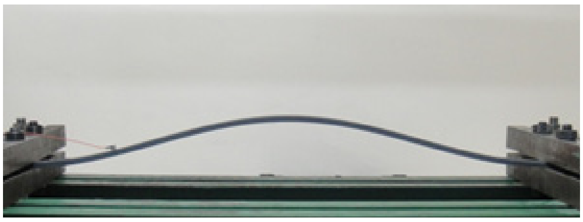

Figure 1. Experimental setup

As can be seen from the table 1, the DQ results are in very good agreement with the FEM ones. The DQ results used only 17 points to model the beam. The experimental natural frequencies of the clamped-clamped beam are compared with those of the DQ method in Table 2, showing good agreement.

The variation of the first four natural frequencies of a clamped-clamped beam in terms of the applied load is computed numerically and demonstrated in Fig. 2. For clamped-clamped beams, the odd frequencies increase rapidly just after the buckling while the even modes experience some decrement after the buckling. Moreover, when the beam undergoes large deflection after the buckling, the second, third and fourth natural frequencies decrease as the applied compressive load increases due to overall stiffness drop caused by the negative geometric stiffness.

The fundamental frequency experiences a sharp growth immediately after the buckling load, and smoothly continues increasing with the increase in applied load. This is because the dynamic stretching-induced stiffness dominates the elastic bending stiffness, while for the other modes the bending stiffness is dominant.

Table 2. Result of spherical vessel without temperature difference with different parameters

Nat. Freq. (Hz)	P/P_{cr}=1.088, End-shortening=130 mm		
	DQM	Experiment	Error (%)
ω_1	95.131	97.500	2.43
ω_2	54.048	55.063	1.85
ω_3	171.43	175.44	2.29

Therefore, the first natural frequency increases with the increase in applied load. Fig. 3 shows the variation of the first 13 natural frequencies of the simple supported beam in terms of the compressive load. As the applied load increases from zero to the buckling load, all the frequencies are reduced smoothly. The same conclusion can be made after buckling, except for the 1st and 11th modes. The fundamental frequency changes rapidly with the increase in applied load; where for a compressive load of $2P_{cr}$, it exceeds the 13th frequency. The first mode shape of the beam is the first symmetric stretching-bending mode, whereas the

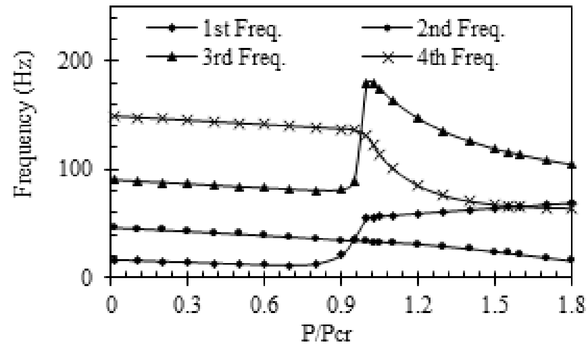

Figure 2. The first four natural frequencies of clamped-clamped beam

Table 1. Natural frequencies of the post-buckled clamped-clamped beam

Nat. Freq. (Hz)	P/P_{cr}=1.2			P/P_{cr}=1.4			P/P_{cr}=1.6		
	DQM	FEM	Error (%)	DQM	FEM	Error (%)	DQM	FEM	Error (%)
ω_1	59.297	59.229	0.114	62.691	62.637	0.086	65.429	65.429	0.000
ω_2	30.898	30.905	0.023	26.519	26.550	0.114	21.086	21.171	0.401
ω_3	146.837	146.998	0.110	126.002	126.062	0.047	113.261	113.380	0.010
ω_4	84.673	84.854	0.214	70.406	70.433	0.039	65.817	65.740	0.118

next nine modes are the bending mode shapes. The 11th mode is the second antisymmetric stretching-bending mode shape. Since the dynamic stretching-induced stiffness dominates the elastic bending stiffness, for the 1st and 11th modes the natural frequencies increase with the increase in applied load.

4- Conclusions

The free vibration of a post-buckled elastic beam was investigated. The solution of the nonlinear differential equations of the beam consists of static and dynamic parts. The differential quadrature method along with an arc length strategy was used to solve the static part, while the same method was utilized to solve the linearized dynamic part. Several numerical as well as experimental case studies on the post-buckled simply supported and clamped-clamped beams were performed.

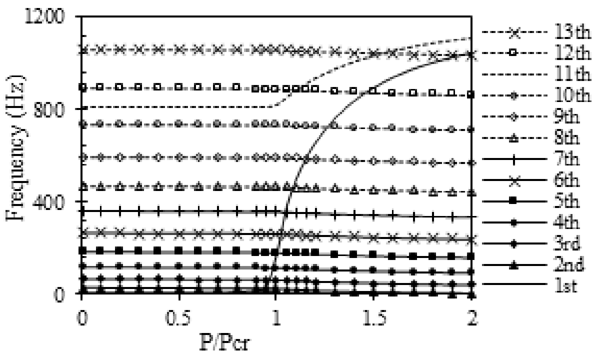

Figure 3. The first 13 natural frequencies of simply supported beam

The results show that the natural frequencies and the mode shapes of the beam can be predicted well by the presented method. The investigation also showed that the boundary conditions as well as the amount of the applied compressive load highly affect the dynamic response of the structure.

It is shown that for clamped-clamped beam, the odd frequencies increase rapidly just after the buckling while the even modes experience a smooth decrement after the buckling. Moreover, when the beam undergoes large deflection after the buckling, the second, third and fourth natural frequencies decrease as the applied compressive load increases, while the first frequency increases. The situation is totally different for the simply supported beams. All the first 13 natural frequencies except the first and eleventh ones decrease as the applied load increases.

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