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Analytical, Numerical and Experimental Investigation of Added mass Influence on Natural Frequencies of an Underwater Structure

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ABSTRACT

It is known that natural frequencies of a structure submerged in water are less than those measured in vacuum. This is due to the effect of added mass of water. For marine structures, fluid inertial (added mass) effects on structural vibration could not be neglected. This paper focuses on analytical, experimental and numerical investigation of natural frequencies of a submerged stiffened plate. The analytical approach is based on the deflection equation of the submerged stiffened plate, the Laplace equation and the Rayleigh method in vibration analysis, considering small oscillations induced by the plate vibration in an incompressible and inviscid fluid. The velocity potential and the Bernoulli equation are adopted to express the fluid pressure acting on the structure. The natural frequencies of the stiffened plate are also practically obtained by using the Fast Fourier Transformation (FFT) in the experimental analysis. The experimental results demonstrate the validity of the numerical and analytical solution. The experimental results validate the derived formulation for the natural frequency of the plate vibrating underwater.

KEYWORDS:

Orthotropic plate, Rayleigh's method, added mass, experimental analysis

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1- Introduction

plates are used in a wide range of engineering applications such as modern construction engineering, aerospace and aeronautical industries, aircraft construction, ship building, and components of nuclear power plants. effect of surrounding medium on vibration of the plates and shells is of primary interest to scientists and engineers working in aerospace, marine and reactor technology [1]. It is therefore very important that the static and dynamic behavior of the plates be clearly understood when subjected to different loading conditions so that they may be safely used in industrial applications. It is well known that natural frequencies of structures in contact with fluid are different from those in vacuum [1]. Therefore, prediction of the natural frequency changes due to the presence of the fluid is necessary in design of structures which are in contact with or immersed in fluid. In general, the effect of the fluid force on the structure is represented as added mass, which lowers the natural frequencies of the structure compared to what that would be measured in vacuum. This reduction in the natural frequencies of the fluid-structure system is caused by increasement of the kinetic energy of the coupled system without corresponding enhancement in the strain energy [1].

In this note, experimental, analytical and numerical analysis for a submerged stiffened plate are studied. The numerical and analytical results are verified compared to the experimental results. This paper is organized as follows: In section two, the analytical solution is derived for the stiffened plate on the water and air. In section three, numerical simulation results for two cases, being in air and fluid are given. The Experimental results are presented in section four. Finally Results and Conclusions are presented in section five.

2- Theoretical Analysis

In this section, theoretical analysis of the stiffened plate is studied in the vacuum and fluid presence. In the analytical study, a stiffened plate undergoing a flexural bending vibration in a body of homogeneous, incompressible inviscid fluid with irrotational flow, is considered. The governing equation for the surface displacement of the plate- fluid system is derived. The boundary conditions for the plate are considered as fix for all sides.

The plate undergoes small amplitude free bending vibration. The fluid motion due to the vibration of the plate produces dynamic pressures $P_L(x,y,t)$ and $P_U(x,y,t)$ on the lower and upper fluid-plate interfaces SL and SU respectively.

Governing equation of submerged deflection of the stiffened plate is given in Eq.(1).

$$D_x\frac{\partial^4 W(x,y,t)}{\partial x^4}+2H\frac{\partial^4 W(x,y,t)}{\partial x^2\partial y^2}+D_y\frac{\partial^4 W(x,y,t)}{\partial y^4}+\rho\frac{\partial^2 W}{\partial t^2}= P_L(x,y,t)-P_U(x,y,t) \qquad (1)$$

where W(x,y,t) is the upward displacement of the plate measured from its static equilibrium position and ρ is density of the plate. D_x and D_y are the flexural rigidities of an orthotropic plate [2].

$H = D_{xy} + 2D_s$, $D_s = \frac{Gh^3}{12}$ and D_{xy} is torsional rigidities for stiffener with rectangular cross section.

The fluid of density ρ_f is assumed to be homogeneous, incompressible, inviscid and irrotational. Therefore, the velocity potential φ(x,y,z,t) satisfies the Laplace equation given by:

$$\nabla^2\phi(x,y,z,t)=0 \qquad (2)$$

Since the fluid motion is irrotational , the unsteady Bernoulli equation is applied as Eq.(3).

$$\rho_f\frac{\partial\phi}{\partial t}+P+\frac{1}{2}\rho_f(\phi_x^2+\phi_y^2+\phi_z^2)+\rho_f g=0 \qquad (3)$$

After combination of the boundary conditions,the plate and fluid equations, the dynamic equation of the plate underwater can be obtained.

$$(\rho+\rho^*)\overline{W}\ddot{T}+D_x\frac{\partial^4 W(x,y,t)}{\partial x^4}+2H\frac{\partial^4 W(x,y,t)}{\partial x^2\partial y^2}+ D_y\frac{\partial^4 W(x,y,t)}{\partial y^4}=0 \qquad (4)$$

In this case we have the following basic equation:

$$\omega_{fluid}=\omega_{air}\frac{1}{\sqrt{1+\frac{m^*}{m}}} \qquad (5)$$

3- Vacuum Case

In this section, the Rayleigh method is used computation of the natural frequencies in the vacuum environment. The Rayleigh method is based on following relation [3, 4].

$$U_{\max}=K_{\max} \qquad (6)$$

By satisfying the boundary conditions of fixed four sides, the deflection of plate can be obtained as follow and this leads to derivation of the natural frequency as given in Eq .8.

$$W = A_{m}\ (1-\cos\frac{2m\pi x}{a})(1-\cos\frac{2n\pi y}{b}) \tag{7}$$

$$\omega_{air} = \frac{2.9}{a^2}\sqrt{\frac{1}{\rho}\left[D_x + D_y(\frac{a}{b})^4 + \frac{2}{3}H(\frac{a}{b})^2\right]} \tag{8}$$

4- Submerged Case

By using the Rayleigh's method for submerged stiffened plate leads to the following natural frequency.

$$\omega_{fluid} = \frac{2.9}{a^2}\sqrt{\frac{m}{\rho(m+m_a)}\left[D_x + D_y(\frac{a}{b})^4 + \frac{2}{3}H(\frac{a}{b})^2\right]} \tag{9}$$

5- Simulation

In this section, the numerical analysis of the stiffened plate is studied in the vacuum and fluid. The boundary conditions for the plate are considered free for all sides. In the modeling of the submerged plate, the shell elements are used to design the plate and the stiffeners. To achieve the natural frequencies of the model, the free vibration analysis is used without any external excitations.

6- Experimental Analysis

The natural frequencies of the stiffened plate are obtained practically by using the modal analysis of the frequency response functions (FRF). Using an FFT (Fast Fourier Transformation) analyzer, the transfer function of the structure can be determined from a force pulse generated by the impact of a hammer and the response signal measured with an accelerometer. By using the FFT and phase analysis the natural frequencies can be easily obtained. The boundary conditions of stiffened plate are considered free on all sides. Thickness of stiffeners and plate are considered to be 4mm. FFT graphs for free vibration of the plate in air and underwater are shown in Fig. 1 and Fig. 2 respectively.

7- Results And Discussion

In order to analyze the stiffened plate in the air, each of accelerometers is connected to separate channels of the PVAT system analyzer. Some tests were done in different conditions.

To evaluate the coupled acoustic analysis in the numerical solution the final solution was compared to the experimental results. The results and the relative error of finite element and analytical solutions compared to the experiment are shown in Table 1.

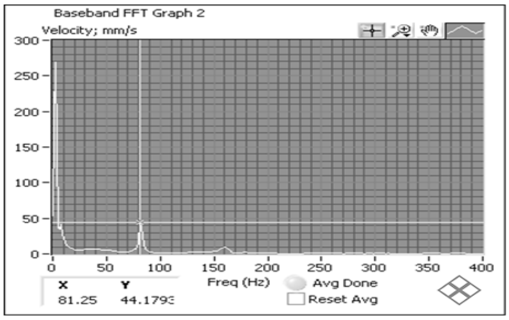

Figure 1. FFT graph in air

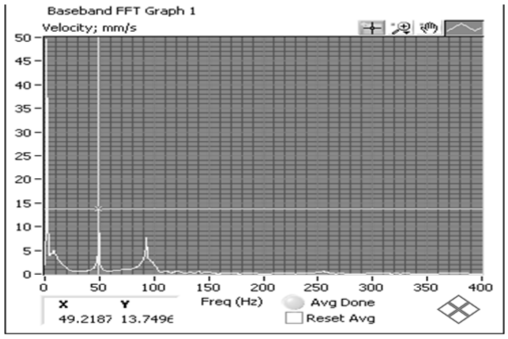

Figure 2. FFT graph underwater

8- Conclusion

This study focuses on analytical, experimental and numerical investigation of natural frequencies of a submerged stiffened plate. The experimental results demonstrate the validity of the numerical and analytical solution. The experimental results validate the derived formulation for the natural frequency of

Table 1. Comparison of the analytical and the numerical results (*cycles/time*)

	Fix boundary conditions		Free boundary conditions	
	simulation	analytical	test	simulation
Natural frequency in air	1147/5	1295	78	83/526
			155	139/24
Natural frequency underwater	618/88	649/026	49/2	45/957
			93/7	91/670

the plate vibrating underwater. Obtained results show that the natural frequencies of a structure submerged in water are less than those measured in vacuum.

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