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Analytical Solution of Non-Linear Free Vibration of Thin Rectangular Plates with Various Boundary Conditions Based on Non-Local Theory

G.A. Varzandian[1]*, S. Ziaei[2]

1- Ph.D. Student of Mechanical Engineering, Yasouj University, Yasouj, Iran
2- Assistant professor, Yasouj University, Yasouj, Iran

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ABSTRACT

This article deals with small-scale effect on nonlinear free vibration of isotropic thin nano-plate using the nonlocal elasticity plate theory. Formulations are based on the Kirchhoff's plate theory, and von Karman-type nonlinearity is considered in strain displacement relations. To include the small scale and the geometrical nonlinearity effects, the governing differential equations are derived based on the nonlocal elasticity theory in conjunction with the von Karman geometrical model in which the effects of rotary inertia and transverse shear are neglected. With cubic non-linearities, Duffing's equation is solved by elliptic integral and natural frequencies are obtained. Also by means of Jacobi elliptic functions, some analytical solutions for deflection of plate are presented. The efficiency and accuracy of the method are demonstrated by comparing the developed result with those available in literature. The effects of various parameters on the nonlinear vibrations of nanoplates are presented. Occurrence probability of internal resonance in rectangular nanoplate is investigated.

KEYWORDS:

Nonlinear Free Vibration, Nonlocal Plate Theory, Elliptic Integral, Jacobi Elliptic Function, Internal Resonance.

* Corresponding Author, Email: varzandian@gmail.com

1- Brief Introduction

Exceptional mechanical, thermal, and electrical characteristics of nanostructures such as nanotubes, nanowires and nanosheets propose various promising applications in micro/nano electromechanical systems. Among these, nanoplates have aroused intensive interest because of their unique structure and superior properties [1].

Vibration of nanostructures is of great importance in nanotechnology. Understanding vibration behavior of nanostructures is the key step for many MEMS & NEMS devices like oscillators, clocks and sensor devices [2].

Based on Eringen's [3,4] nonlocal elasticity theory, size effects are taken into account by the integration of a scale parameter into classical continuum models. In nonlocal elasticity theory, the stress at a reference point is assumed to be a functional of the strain field at every point in the body.

In this paper, the small-scale effect on the nonlinear free vibration of isotropic thin nano-plate using the nonlocal elasticity plate theory for various combinations of boundary conditions is investigated. Also occurrence probability of internal resonance in rectangular nanoplate is investigated.

2- Methodology

The formulations are based on the Kirchhoff's plate theory, and von Karman-type nonlinearity is considered in strain displacement relations. To include the small scale and the geometrical nonlinearity effects, the governing differential equations are derived based on the nonlocal elasticity theory in conjunction with the von Karman geometrical model in which the effects of rotary inertia and transverse shear are neglected. With cubic non-linearities, Duffing's equation is solved by elliptic integral and natural frequencies are obtained. Also by means of Jacobi elliptic functions, some analytical solutions for deflection of plate are presented. The efficiency and accuracy of the method are demonstrated by comparing the developed result with those available in literature. The effects of various parameters on the nonlinear vibrations of nanoplates are presented. The solution methodology is applied to some kinds of boundary condition as pointed out and the related curves are presented for comparison. By means of amplitude incremental finite element method [5], occurrence probability of internal resonance in rectangular nanoplate is investigated.

3- Numerical Results

Effects of various physical parameters on the nonlinear vibration of nano palte such as scale parameter, aspect ratio and boundary conditions are investigated. Some of the numerical results are represented in Figures 1 and 2.

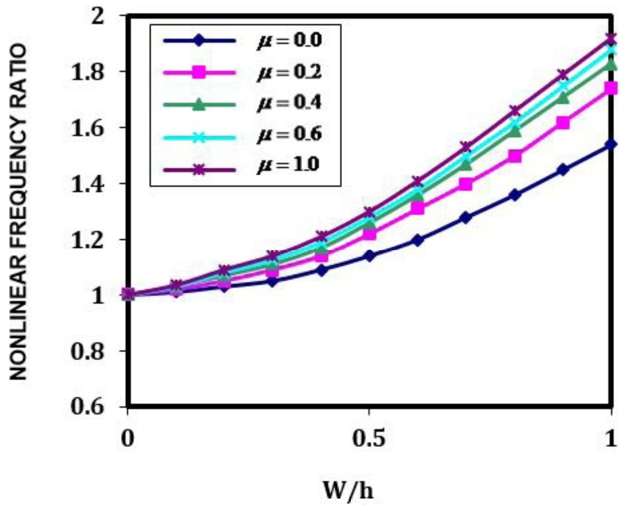

Figure 1. The effect of scale parameter on nonlinear frequency ratio versus dimensionless amplitude curves for squared plate with *a/h*=20

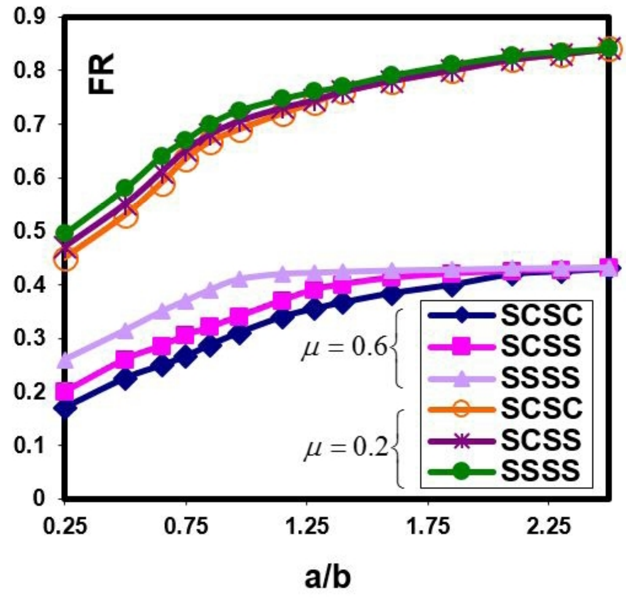

Figure 2. Variations of fundamental frequency ratio with aspect ratio

Figure 1 shows the effects of scale parameter on the nonlinear frequency ratio versus dimensionless amplitude curves for simply supported squared plate with a/h=20 . It is to be noted that the scale factor μ=0 corresponds to the case of classical continuum mechanics model and demonstrates a lower bond solution. It is observed that the small-scale effects have a large impact on the nonlinear frequencies, which shows the necessity of a nonlinear analysis.

To investigate the effects of the aspect ratio on frequency ratio, Figure 2 is plotted. This figure depicts variations of the fundamental frequency ratio with aspect ratio for two specified values of nonlocal parameter μ=0.2, 0.4 and three different boundary conditions. By increasing the aspect ratio, the frequency ratio of a nano-plate with SSSS, SCSS and SCSC boundary conditions, shown in Figure 2, increases. Generally the aspect ratio has more influences on the frequency ratio in the case of stiff boundary conditions (i.e. SCSC, SCSS and SSSS).

4- Conclusions

Some of the important achievements of this research are listed as following:

- A decrease in the dimension of the plate causes the effect of the non-local parameter to become more significant and consequently to decrease the frequency parameter.
- Vibration characteristic of the nano-plate depends on the size of the plate, mode of vibration and non-local parameter.
- Using a local plate model in the frequency analysis of a nano-plate can cause an overestimate.
- The non-local parameter is more effective in higher modes of vibration.
- The non-local effect in a square nano-plate is more significant in comparison with a rectangular nano-plate.
- The effect of the non-local parameter on the vibration behavior is almost independent of the length-to-thickness ratio.
- The frequency ratio decreases with the increasing vibration amplitude and Poisson's ratio, but it decreases with the decreasing thickness ratio and aspect ratio.
- The small-scale parameter reduces the natural frequencies, but increases the nonlinear-to-linear frequency ratios of the nano-plates. In other words, the local theories overestimate the natural frequencies.
- The small-scale effects have more significant influence on the higher order frequencies.

5- Main References

[1] Setoodeh AR, Malekzadeh P, Vosoughi AR., 2011. "Nonlinear free vibration of orthotropic graphene sheets using nonlocal mindlin plate theory", *Proc Inst Mech Eng, Part C: J Mech Eng Sci*, 226, No.7, pp. 1896-1906.

[2] Pradhan SC, Phadikar JK., 2009. "Nonlocal elasticity theory for vibration of nanoplates", *J. Sound Vib.*, 325, pp. 206–23.

[3] Eringen, A.C., 1983. "On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves", *Journal of Applied Physics.*, 54, pp. 4703–4710.

[4] Eringen, A.C., 2002. "*Nonlocal Continuum Field Theories*", Springer, New York.

[5] Lau, S. L., Cheung, Y. K., and Wu, S. Y., 1984. "Internal Resonance by Amplitude Incremental Finite Element", *ASME JOURNAL OF APPLIED MECHANICS*, 51, pp. 845-851.