



# Critical Thickness of a Porous Layer with Respect to Porosity and Its Effect on Heat Transfer Rate

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**ABSTRACT:** Investigation of thermal effects of a porous layer in an enclosure has not been completely studied in the literature and this challenge is generally considered to be an open research topic that may require more study. As heat transfer rate in a vertical porous layer is higher than a horizontal layer, in this paper effect of thickness of the vertical porous layer on natural convection in an enclosure is completely investigated using the lattice Boltzmann method. Therefore, after studying effect of position of a porous layer on the Nusselt number, the effect of thickness of the porous medium on heat transfer rate is investigated. The obtained results show that a considerable amount of convective heat rate is transferred using the middle-vertical porous layer; and by increasing the thickness of the layer, the values of Nusselt number decrease which can be described by definition of the modified Rayleigh number. In addition, the critical thickness of the porous layer inside the cavity is reported in the moderate Rayleigh numbers for the first time.

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**1- Introduction**

Due to widespread applications of porous medium in many fields of engineering, several experimental and theoretical investigations have been carried out to investigate convection heat transfer in a porous enclosure. In these studies, various models such as the Darcy, the Brinkman-extended Darcy, and the Forchheimer-extended Darcy models have been used to examine fluid flow in porous media. Recently, a generalized model in modeling flow in porous media is so-called developed general Navier-Stokes model (Brinkmann-Forchheimer extended Darcy model) has been presented, in which all fluid forces and the solid drag force are considered in momentum equation [1].

In this paper LBM base on developed general Navier-Stokes model is used to simulate natural convection heat transfer in a cavity partially filled with a vertical porous layer. As mentioned before, this problem has attracted attention for design of electronic cooling. For this purpose, after examining the effect of position of vertical porous layer on the average Nusselt number, the effect of the effective dimensionless parameters such as Rayleigh number, Darcy number and the porosity and especially the effects of the thickness of the porous layers on the fluid flow and heat transfer have also been investigated to find optimal heat transfer rate.

**2- Governing Equations**

The present study is concerned with natural convection in an  $L \times L$  square enclosure with vertical walls held at uniform temperature  $T_h$  and  $T_c$  ( $T_h > T_c$ ) while connecting horizontal walls are insulated. As shown in Fig. 1, the enclosure is partially filled with a vertical layer of a porous material with

the thickness of  $S$ . The location of the porous layer relative to the hot wall ( $Z$ ) is shown in Fig. 1.

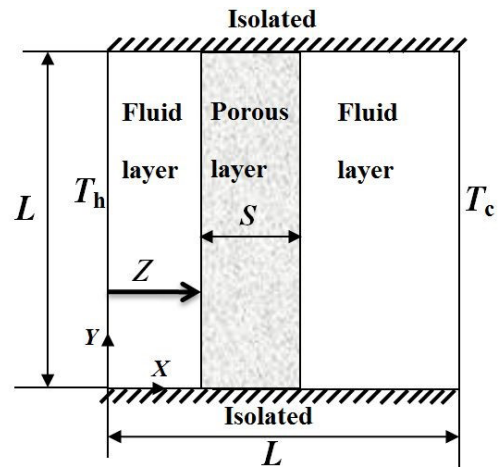


Figure 1. Schematic of the cavity with a vertical porous layer

The continuity equations, the Brinkman-Forchheimer equation and energy equation for incompressible fluid flow and convection heat transfer in porous media at representative elementary volume (REV)-scale are as follows:

$$\nabla \cdot \vec{u} = 0 \tag{1}$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \left( \frac{\vec{u}}{\varepsilon} \right) = - \frac{1}{\rho_f} \nabla (\varepsilon p) + \nu_e \nabla^2 \vec{u} + \vec{F} \tag{2}$$

$$\sigma \frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T = \vec{\nabla} \cdot (\alpha_m \vec{\nabla} T) \tag{3}$$

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where  $u$ ,  $p$  and  $T$  are the volume-averaged velocity, pressure and temperature of the fluid, respectively.  $v_e$  is an effective viscosity parameter,  $\varepsilon$  is the porosity of the medium and  $\alpha_m$  is the thermal diffusivity. The coefficient  $\sigma$  represents the ratio between the heat capacities of the solid and fluid phases, i.e.  $\sigma = \varepsilon + ((1-\varepsilon)\rho_s C_{ps})/(\rho_f C_{pf})$  where  $\rho_f$  and  $\rho_s$  are the fluid and solid densities,  $C_{pf}$  and  $C_{ps}$  are the fluid and solid specific heats at constant pressure, respectively. The total body force  $F$  due to the presence of a porous medium and other external force fields in Equation (2) is expressed as:

$$F = -\frac{\varepsilon v}{K} \mathbf{u} - \frac{\varepsilon F_\varepsilon}{\sqrt{K}} |\mathbf{u}| \mathbf{u} + \varepsilon \mathbf{G} \quad (4)$$

$$\mathbf{G} = -g\beta(T - T_0) + a \quad (5)$$

where  $v$  is the viscosity of the fluid and  $G$  is given by The first term in Equation (5) represents the buoyancy force, and the second term is the acceleration due to other external force fields. The geometric function  $F_\varepsilon$  and the permeability  $K$  of the porous medium are related to the porosity  $\varepsilon$ , based on Ergun's experimental investigations.

Equations (1-3) are characterized by some dimensionless parameters: the Darcy number ( $Da = K/L^2$ ), the Prandtl number ( $Pr = \nu/\alpha$ ), the Rayleigh number ( $Ra = g\beta(Th - Tc)/\nu\alpha$ ) and the viscosity ratio ( $Je = \nu_e/\nu$ ). In addition, other dimensionless parameters are: the dimensionless temperature ( $\theta = (T - Tc)/(Th - Tc)$ ), the dimensionless velocity in x-direction ( $u^* = uL/\alpha$ ) and the dimensionless velocity in y-direction ( $v^* = vL/\alpha$ ). The two important dimensionless variables for analysis of heat transfer in natural convection inside a porous enclosure are modified Rayleigh number or Darcy-Rayleigh number ( $Ra_m$ ) and average Nusselt number. The Darcy-Rayleigh number can be expressed in terms of the fluid

$$Ra_m = Ra \cdot Da \quad (6)$$

Rayleigh number as:

The average Nusselt number throughout the cavity is

$$\overline{Nu} = \frac{1}{L(T_h - T_c)} \int_0^L \left( \int_0^L \frac{uT}{\chi} - \frac{\partial T}{\partial x} dy \right) dx \quad (7)$$

expressed as:

where the binary parameters  $\chi$  is the thermal diffusivity of the medium.

### 3- Numerical Method

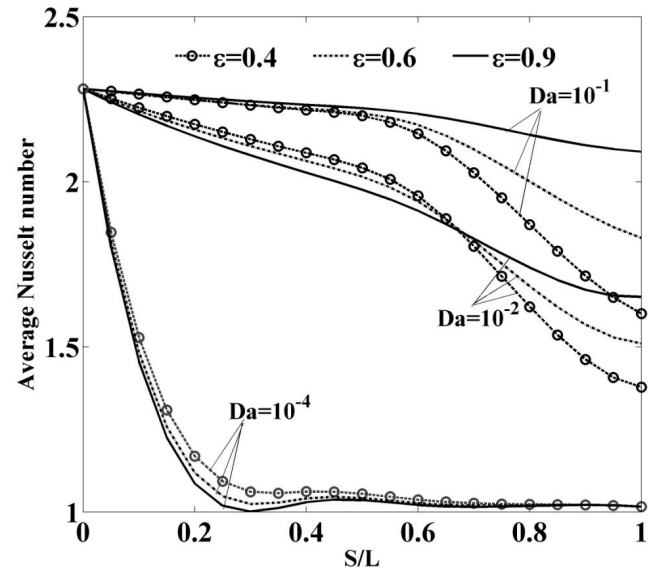
In this paper, the lattice Boltzmann method is used for numerical simulations. This method provides an alternative way to solve the partial differential equations for two dimensional incompressible thermal flows through porous media. A thermal lattice Boltzmann scheme is used, which solves the discrete equation of density and internal energy distribution function [2].

### 4- Results

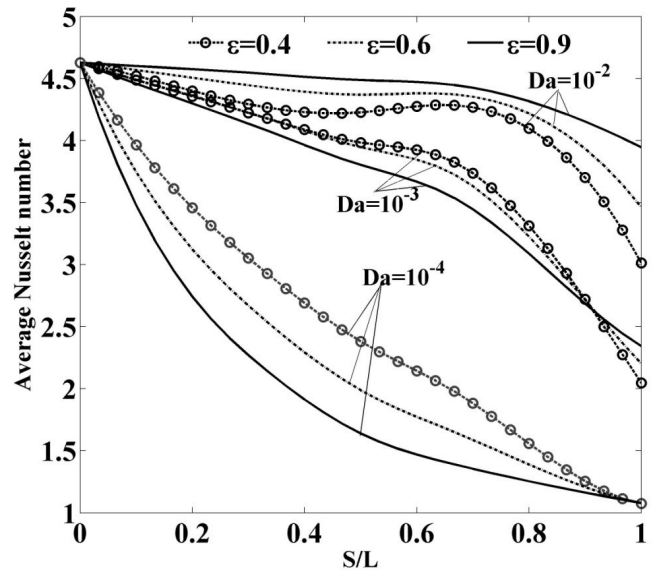
The effect of porous layer thickness on the average Nusselt number for various values of Darcy, Rayleigh and the porosity of the porous layer are shown in Fig. 2. As depicted in Fig. 2, for a fixed porous layer thickness ( $S/L$ ), at high modified Rayleigh numbers ( $Ra=10^5$ ,  $Da=10^{-2}$ ), the convection heat transfer mechanism is dominated, therefore, as the porosity

of the porous layer increases, the average Nusselt number increases. In contrast, for a small modified Rayleigh numbers the trends are reversed ( $Ra=10^4$ ,  $Da=10^{-4}$  and  $Ra=10^5$ ,  $Da=10^{-4}$ ). In other word, for a large value of modified Rayleigh number, the effect of porosity on the Nusselt number is different from the case of small modified Rayleigh number. In this case, the dominant mechanism of heat transfer may be similar to conduction mechanism, therefore as the porosity of the porous layer increases, the heat transfer surface between the fluid and solid matrix decreases so the rate of heat transfer decreases. It is noted that for a moderate modified Rayleigh number ( $Ra=10^5$ ,  $Da=10^{-3}$  or  $Ra=10^4$ ,  $Da=10^{-2}$ ), there is a critical porous thickness, i.e.  $(S/L)_{cr}$ .

For the thicknesses larger than the  $(S/L)_{cr}$ , increasing the porosity of the layer leads to increasing in the heat transfer, but for thickness smaller than the  $(S/L)_{cr}$ , increasing the porosity results in a decrease in the value of Nusselt number.



(a)



(b)

Figure 2. The effect of porous layer thickness ( $S/L$ ) on the average Nusselt number, (a)  $Ra=10^4$ , (b)  $Ra=10^5$

### 5- Conclusion

The lattice Boltzmann model was used to simulate flow and temperature fields arise due natural convection heat transfer in a square enclosure containing a vertical porous layer. The effects of non-dimensional parameters such as  $Da$ ,  $Ra$ ,  $\varepsilon$  and the thickness ( $S/L$ ) of porous layer on the flow pattern as well as heat transfer were completely investigated.

### References

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