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Vibration Behavior and Crack Detection of a Cracked Short Beam Under a Axial Load

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ABSTRACT

In this research, firstly, vibration behavior of a cracked short cantilever beam under axial load is investigated, and then, an analytical approach is proposed for crack identification based on the vibration. The cracked section of the beam is considered to be a flexible element, which divides the beam into two segments. Using fracture mechanics theory, the local flexibility of the crack is modeled as a mass-less tensional spring. By applying boundary conditions and inner conditions at the crack location, and taking into account the effects of shear deformation and rotary inertia, governing equation of motion for the cracked beam is derived. The influence of the axial load and the crack parameters on the vibration behavior of the cracked beam are studied by establishing and solving the corresponding eigenvalue problem directly. Then, in order to predict the crack depth and location through the known natural frequencies of the cracked beam, which are obtained through the experimental tests, the corresponding inverse problem is established and solved analytically. The results have been validated by experimental and theoretical data reported in the literatures.

KEYWORDS:

Cracked beam, Axial load, Timoshenko beam, Crack detection.

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1- Introduction

Studies show that vibration analysis is an efficient method for crack detection in structures. In this method, evaluating the changes in dynamic properties may lead to the crack intensity and location.

Many researchers studied the crack detection based on the vibration analysis for the cracked beams [1]. In many of these studies, the effect of an axial load applied on the beam is neglected. Such an assumption can reduce the accuracy of the model. Recently, Mei et al. [2] investigated the vibration behavior of an axially loaded cracked Timoshenko beam from the wave standpoint. Crack detection is not investigated in their studies.

In the present study, the vibration analysis method is used for the crack detection in an axially loaded cracked Timoshenko beam. The crack is modeled as a mass-less torsional spring, in which the cracked beam is separated into two intact segments. The governing differential equation is derived by the Hamilton's principle and the methods for solving both the forward problem (determination of frequencies of the beam knowing the crack parameters) and the inverse problem (determination of the crack parameters through the natural frequencies) included. The theoretical results are verified by experimental data reported in the literature.

2- Modeling Of Axially Loaded Cracked Beam

In this study, lateral free vibration of an axially loaded cantilever cracked Timoshenko beam is investigated (Figure 1). The stiffness of the torsional mass-less spring modeling the crack, k_t, is given in Eq. (1) [1].

$$k_t = \frac{Ewh^4}{72\pi \int_0^a a\left[f\left(\frac{a}{h}\right)\right]^2 da} \tag{1}$$

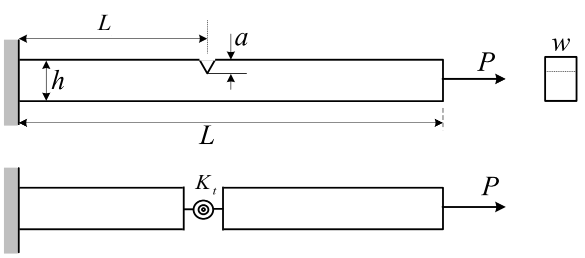

Figure 1. Axially loaded cracked Timoshenko beam

Using the Hamilton's principle, the governing equation of an axially loaded Timoshenko beam is obtained as Eqs. (2) and (3).

$$EI\frac{\partial^2\psi(x,t)}{\partial x^2} + \kappa GA\left[\frac{\partial y(x,t)}{\partial x} - \psi(x,t)\right] - \rho I\frac{\partial^2\psi(x,t)}{\partial x^2} = 0 \tag{2}$$

$$\rho A\frac{\partial^2 y(x,t)}{\partial t^2} - \kappa GA\left[\frac{\partial^2 y(x,t)}{\partial x^2} - \frac{\partial\psi(x,t)}{\partial x}\right] - P\frac{\partial^2 y(x,t)}{\partial x^2} = 0 \tag{3}$$

where, y is the transverse deflection, ψ is the angle of rotation due to bending moment, E is the modulus of elasticity, G is the modulus of rigidity, A is the area of cross-section, I is the area moment of inertia and is the axial load.

In absence of the axial load ($P = 0$), the above equations reduce to the same ones reported in [2]. Using the Hamilton's principle, the associated boundary conditions are obtained as:

$$\left(EI\frac{\partial\Psi}{\partial\beta}\right)\delta\Psi\Big|_{\beta=0}^{1} + K_\theta\frac{dY}{d\beta}\Big|_{\beta-0} = 0 \tag{4}$$

$$\left\{\frac{L^2}{EI}\left(\frac{d^2\Psi}{d\beta^2} + r^2b^2\Psi + \frac{\varphi}{L}\frac{dY}{d\beta}\right)\right\}\delta Y\Big|_{\beta=0}^{1} + K_{te}Y\big|_{\beta=0} = 0. \tag{5}$$

The conditions for the continuity of the displacement field, the moment and the shear force at the crack location, and the relation between the slopes at the two sides of the crack can be written as:

$$Y_1(\beta) = Y_2(\beta) \tag{6}$$

$$\frac{d\psi_1(\beta)}{d\beta} = \frac{d\psi_2(\beta)}{d\beta} \tag{7}$$

$$\frac{d^2\Psi_1(\beta)}{d\beta^2} + r^2b^2\Psi_1(\beta) + \frac{\varphi}{L}\frac{dY_1}{d\beta}(\beta) = \frac{d^2\Psi_2(\beta)}{d\beta^2} + r^2b^2\Psi_2(\beta) + \frac{\varphi}{L}\frac{dY_2}{d\beta}(\beta) \tag{8}$$

$$\frac{dY_1(\beta)}{d\beta} + \frac{L}{k}\frac{d\psi_1(\beta)}{d\beta} = \frac{dY_2(\beta)}{d\beta} \tag{9}$$

3- Natural Frequencies

From the boundary conditions and the compatibility Eqs. (8) to (13) and applying the governing equations, the characteristic equation is obtained as Eq. (14).

$$\det(\Delta) = 0 \tag{10}$$

4- Crack Identification

Knowing the natural frequencies obtained from the experiments, one can solve the inverse problem to determine the crack parameters. For a given natural frequency, Eq. (10) represents the relation between the crack location and the stiffness of the torsional spring which models the crack. Since the torsional spring stiffness, k_t , and the crack location is assumed to be independent of the vibration mode, therefore by knowing the three first natural frequencies of the cracked beam, one can plot the variation of the stiffness k_t with respect to the crack location β for each mode. The intersection of the three curves will indicate the possible crack location and magnitude of the stiffness.

5- Results and Discussion

To make a comparison between the proposed method and the results reported in the literature, the lateral vibration behavior of a cantilever cracked beam is discussed. The natural frequencies of a cracked beam for various crack parameters and axial loads are obtained and compared with the experimental data.

There is a good agreement between the results obtained through the proposed method and the experimental data reported in Lele and Maiti study [3]. The maximum error for the prediction of the natural frequencies is less than 8.5%.

6- The Inverse Problem

The method for the crack detection is tested for several combinations of crack depths and locations. In Figure 2, the variation of torsional spring constant, k_t, against the crack location are plotted for the three first vibration modes of the cracked beam. The horizontal and vertical coordinates of the intersection point of these three curves correspond to the crack location and torsional spring stiffness. Via Eq. (1) and knowing the spring stiffness, the crack depth can be obtained.

In order to verify the accuracy of the model, the same axially loaded cracked cantilever beam, which has been analyzed in Mei et al. study [2], is considered. The cracked beam with a relative depth of 0.3 and three different relative crack locations of 0.3, 0.5 and 0.8 is studied. Table 1 shows the natural frequencies of the beam for various axial loads as well as the predicted parameters of the crack. As it is seen in Table 1, the results are in good agreement with those ones reported in results of Mei et al. study [2]. The last two columns of Table 1 reveal that the error in the prediction of the crack location and depth are less than 6.7% and 14%, respectively.

Table 1. Predicted crack location and depth for cracked beam

Axial force, $P(N)$	Crack location, β^*	Crack depth, α	Natural Frequencies [3]			Predicted data		Error in β^* (%)	Error in α (%)
			f_1	f_2	f_3	β^*	α		
15	0.3	0.3	46.0	225.0	592.8	0.31	0.32	3.33	6.71
0	0.3	0.3	34.1	215.2	571.2	0.28	0.26	6.67	13.12
15	0.5	0.5	46.8	217.1	614.3	0.51	0.43	2.00	14.00
0	0.5	0.5	34.9	207.0	592.5	0.53	0.55	6.01	10.00

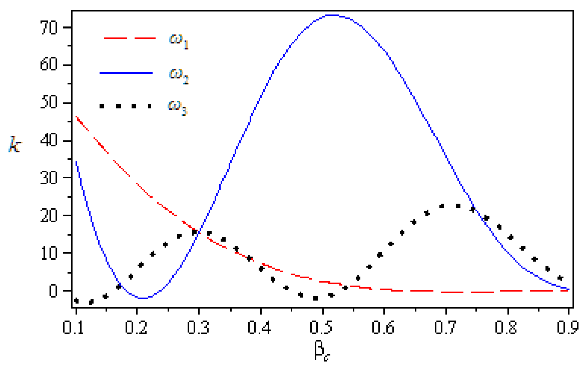

Figure 2. Variation of stiffness of torsional spring constant with respect to the crack location

7- Concluding Remarks

In this paper, the vibration behavior and the crack detection problem of an axially loaded cracked Timoshenko beam are investigated. The results show that:

- The presence of the crack causes the local stiffness at the crack location to be reduced, and for a given crack location, the deeper cracks affect the vibration characteristics more seriously than the smaller ones.
- For a given crack depth, the maximum reduction

in the beam natural frequencies, due to the crack occurs, when the crack locates at a point with a maximum bending moment. Therefore, as the crack approaches to the nodal points of the beam at a certain vibration mode, its influence on the corresponding natural frequency becomes negligible.

- For a given crack location and depth, the tensile axial load reduces the effect of the crack on the beam natural frequencies. Likewise, the tensile axial load increases the natural frequencies.
- Applying the proposed inverse problem, the maximum error in the prediction of the crack depth and location are less than 14% and 7%, respectively

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