



Computation of Permissible Perturbations in Dynamic Parameters of an Articulated Vehicle to Maintain Stability

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ABSTRACT: The aim of this research is to compute allowable ranges of uncertain parameters of tires of an articulated vehicle, that affects tire torques and forces, to maintain stability. A seven-degree-of-freedom model of the vehicle is adopted while moves on a straight track. A nominal output feedback controller is designed for the model of the vehicle with nominal parameters, in order to attain acceptable performance in presence of external disturbances. Due multi-linear uncertainty structure of characteristic equation of the linearized model of the vehicle, the polynomial method is used to compute the stability margins for the uncertain parameters. The computed margins are verified by plotting the root-locus of the closed-loop poles of the system when the parameters are perturbed inside the computed margins. Also, a more realistic model of an articulated vehicle is built in the environment of ADAMS software to verify the computed stability margins. It is observed that the computed bounds of parameter perturbations are relatively exact and the perturbations inside the computed ranges do not destabilize the vehicle.

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1- Introduction

An articulated vehicle consists of two or more separate bodies that are joined to form a more maneuverable vehicle. The most common unstable mode of motion of these vehicles is the oscillatory motion of rear and front parts. Effects of several structural properties of the vehicle on its stability have already been studied by several researchers [1,2]. In this paper, a linear model of an articulated vehicle was used from the previous work of Azad et al. [2]. For nominal condition of tire parameters an output feedback controller is designed to reduce the effects of disturbances on articulation angle. Then, robust stability margins for the tire parameters are computed. The characteristic polynomial of this model is multi-linearly dependent on uncertain tire parameters. Using the Zero Exclusion Theorem [3], a method was introduced in Bozorg study [4] to compute the parameter stability margins for such systems.

2- Methodology

A linear model from Azad et al. study [2] is used in this paper to study stability of an articulation vehicle under tire uncertain parameters. In this model, vehicle is faced to small perturbation and travels on flat road with constant velocity. The tire longitudinal forces are considered as a linear function of the longitudinal slip:

$$F_x = C_s S \quad (1)$$

where, C_s is the longitudinal slip stiffness. Also, the tire lateral forces and the aligning moment are considered as linear functions of the slip angle:

$$F_y = C_a \alpha \quad (2)$$

$$M_z = -C_T \alpha \quad (3)$$

where, C_a is the lateral slip stiffness and C_T is the aligning moment stiffness. Suppose that this slip stiffness is equal for both (right and left) wheels. The equation of motion, in the form of state variables, is:

$$M\dot{x} = -Ex + FU(t) \quad (4)$$

where the state variable vector is:

$$x = [\dot{\psi}, v, \phi, \dot{\phi}, \dot{\theta}_r, \dot{\theta}_l] \quad (5)$$

In Equation (5), $\dot{\psi}$ is the yaw rate of the front body, v is the lateral velocity, ϕ is the relative angle between the front and the rear body, $\dot{\theta}_r$ is the relative rotational velocities of the right and the left wheels of front body and $\dot{\theta}_l$ is defined for rear body as $\dot{\theta}_r$. In Equation (4), the input is the torque (ΔT) applied to the rear wheels of the vehicle. This torque is used to control tire forces in disturbances and oscillatory motion of rear and front parts [2]. The tire force slip stiffness is perturbed to different values when the vehicle travels on different surfaces. Also, the stiffness has different values, depending on the models [5]. From [6], the ranges of perturbation of the stiffness for front and rear tires are calculated.

Consider that the nominal values of these parameters are the mean values of their ranges. For vehicle with nominal parameters, an output feedback controller is designed to reduce the effect of disturbance torque applied to the front and the rear body of an articulated vehicle. This torque may

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increase up to 30000 N.m and can be modeled as a pulse function with a limited application time (e.g., 0.2 s). Infinity norm of a time function, introduces its maximum value over the time. To attenuate the road disturbance, this norm must be minimized. In absence of controller, the infinity norm of articulation angle was obtained as about 0.3 rad. Output feedback controller must reduce this value to less than 0.1 rad, while keeping the system stable. For designing this controller, an optimization problem can be defined to search for an optimal controller matrix:

$$\text{Minimize } \|\phi(t, \bar{K}_i)\|_{\infty} \quad (6)$$

Subjected to:

$$\|\phi(t, \bar{K}_i)\|_{\infty} \leq 0.1 \quad (7)$$

and system remains stable. This controller is selected such as to feedback only several simply measurable variables. They include the relative rotational velocity of right and left wheels and the relative yaw rate motion of the front and the rear bodies. This optimization problem was solved by MATLAB software. The optimal controller matrix was obtained as:

$$\bar{K} = 10^5 \times [0, -1.169, +0.130, 0, -0.016, 0.185] \quad (8)$$

In presence of this optimal controller, infinity norm of articulation angle obtained 0.09 rad. To verify this result, a more realistic model of an articulated vehicle is built in the environment of ADAMS software. The controlled and uncontrolled disturbed vehicle response in articulation angle is plotted in Figure 1.

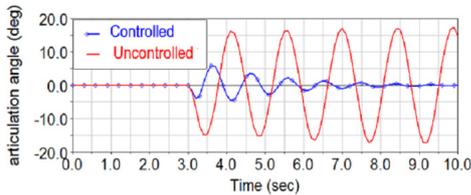


Figure 1. Controlled and uncontrolled disturbed vehicle response in articulation angle in nominal condition

As mentioned in this section, there are six uncertain tire parameters in this model and the uncertainty vector is defined as:

$$q = [C_{af}, C_{ar}, C_{Tf}, C_{Tr}, C_{Sf}, C_{Sr}] \quad (9)$$

There is no guarantee that vehicle is stable in the whole range of parameters perturbation. In this paper, stability margin for uncertain parameters is calculated by polynomial method [4]. The characteristic polynomial of this articulated vehicle are derived as the polynomial:

$$p(q, s) = a_n(q) s^n + \dots + a_1(q) s + a_0 \quad (10)$$

which are multilinear functions of the uncertain parameters. In this method, to compute the stability margins, the ‘Zero Exclusion Theorem’ is used. This theorem states that a polynomial family $p(s, Q)$ is robustly stable, if and only if [3]:

1) There exists a stable polynomial $p(s, q) \in p(s, Q)$

2) $\forall \omega > 0 \rightarrow 0 \notin p(j\omega, Q)$,

3) $\forall q \in Q \rightarrow a_n(q) \neq 0$

For the articulated vehicle under study, the characteristic polynomial is stable at the nominal values of the parameters. The coefficient of the highest degree is constant. So, conditions 1) and 3) are satisfied and only condition 2) must be checked for all frequencies. This search is done by solving an optimization problem [4]. The computation leads to the stability margin radius of 0.627, which occurs at the frequency of 8.74 rad/s.

To verify the results of the polynomial method, the stability of the vehicle inside and outside of the computed margin is checked by a virtual model in ADAMS. As an example, the variation of vehicle’s articulation angle due to initial condition of $\dot{\phi}_0 = 0.6$ rad/s for a corner point of the uncertainty domain:

$$q_1 = [C_{af}^+, C_{ar}^+, C_{Tf}^-, C_{Tr}^-, C_{Sf}^+, C_{Sr}^-] \quad (11)$$

is shown in Figure 2. At this corner q_1 , the instability happens in ADAMS simulations at the radius of 0.69, which is pretty close to the value of 0.627 computed from the polynomial method. So, despite its simplifications, the polynomial method can compute the robust stability margin with a good approximation.

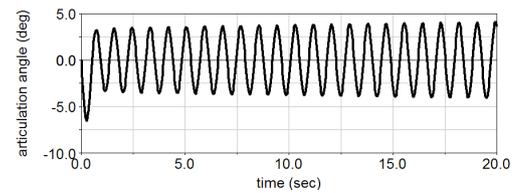


Figure 2. System response in articulation angle to initial condition $\dot{\phi}_0 = 0.6$ rad/s for corner q_1 of uncertain parameter space of radius 0.69

3- Conclusions

In this paper, stability of an articulated vehicle at higher speeds is studied. For a 7 DOF model of this vehicle from [2], the tire parameters are considered as uncertain parameters and their ranges of perturbation are calculated. For nominal condition of these parameters, an output feedback controller is designed. This controller is able to reduce the effects of disturbance in the articulation angles up to 70%. Then, the polynomial results are used to compute the stability margins for the uncertain parameters. The results show that the polynomial method can compute the robust stability margin with an adequate accuracy.

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