# Dynamic Behavior Analysis of Moderately Thick Composite Laminated Plates Containing Square Delamination Using Spline-Finite Strip Method

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**ABSTRACT:** The dynamic behavior of laminated composite panels subjected to in-plane end-loads is investigated using finite strip method. The panel is assumed to contain square delamination. Friction and contact conditions at delaminated interfaces are not considered. It is also assumed that the delamination exists before loading and delamination growth is not considered. A general loading consisted of time varying as well as constant component is assumed and the static buckling, natural frequencies and also parametric instability of the panel have been investigated. The loading assumed to be uniformly distributed throughout the panel area. The dynamic behavior of the delaminated panel has been extracted by using an enhanced B-spline finite strip formulation on the basis of the principle of virtual work. A third order Reddy type shear deformation theory is utilized in order to bring the effects of moderately thick laminate into account. The effects of loading as well as boundary conditions on the dynamic behavior of the structure are studied. The dynamic instability regions are extracted using the Bolotin's first order approximation. In order to demonstrate the capabilities of the developed method in predicting the structural dynamic behavior, some representing results are obtained and compared with those available in the literature.

## 1- Introduction

Aeronautic, space and marine structures are among those main disciplines where the least structural weight besides providing high enough strength must be achieved. Thus, thin-walled structures will usually come to play. A thin-walled panel under in-plane dynamic loading, generally a harmonically varying excitation with a constant frequency added to a constant mean load, meets situations where the instability conditions may appear. The amplitude of the corresponding dynamic instability load may even be lower than the value corresponding to static bifurcation point. These excitation conditions are prevalent in case of mechanical structures as well as fluid-structural interactions. A widespread defect of composite laminated structures is debonding of layers called delamination. Delamination occurrence in causes total strength reduction of the structure and activates low energy local instability and failure modes. Thus, it is of high importance to calculate the stiffness reduction of a delaminated structure. Therefore, it is essential to study the various effects of loading and delamination on the dynamic characteristics and response of the layered plates under static and periodic in-plane loads.

In this paper, the dynamic behavior of moderately thick laminated flat panels containing delamination subjected to uniform in-plane load has been investigated through the application of finite strip method. The loading is assumed to change harmonically with time. A B-spline version of Finite Strip Method (FSM) has been developed. The formulations are based on Reddy type higher order shear deformation theory in order to include the transverse shear stresses effect in case of moderately thick structures. The governing equations are derived using full energy concepts on the basis of the principle of virtual work. The dynamic behavior, including natural frequencies as well as instability load frequency regions corresponding to the assumed in-plane parametric loading utilizing the Bolotin's first order approximation, are extracted. Some representative problems are numerically studied and compared to those in the literature wherever available. To the best of author's knowledge, this is the first application of Spline FSM to the problem.

## 2- Methodology

In case of the existence of an internal delamination, to model the effects of a single delamination, two layers of strips are considered. Fig. 1 depicts the strip knots at the connecting lines of the delamination area. The corresponding knots at the top and bottom strips everywhere are merged together except those placed inside the delamination area. The number of strip knots must be chosen such that just one knot lays on the delamination boundary. Also, the number of strips in the panel width must be selected such that the delamination ends on one of the strips connecting lines. Merging the knots located on the delamination ends guarantees the proper delaminated region physical boundary conditions.

Through applying the principle of virtual work in an energy formulation approach, a system of Mathieu type differential governing equations is obtained for the problem of parametric instability

$$M\ddot{\delta} + (K - a^S K_g^S - a^D \cos \omega t . K_g^D)\delta = 0 \qquad (1)$$

where $M$, $K$, $K_g^S$, and $K_g^D$ are square global structural mass, stiffness, static geometrical, and dynamic geometrical matrices, respectively. $\delta$ is the global vector of model degrees
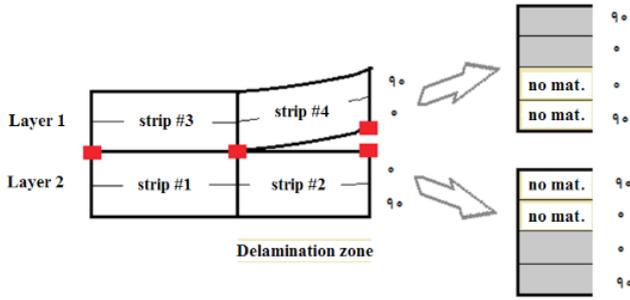
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**Figure 1. An example of a mid-thickness delamination and twin-layer modeling technique**

of freedom. Once the initial stress is totally ignored, Eq. (1) is reduced to free vibration problem whilst in case of excluding the effects of time and mass, under initial stresses, the problem is altered to static buckling one. By implementing Bolotin's first order approximation corresponding to twice the loading period, the time-varying vector of $\delta$ is approximated as:

$$\delta = A\sin\left(\tfrac{1}{2}\omega t\right) + B\cos\left(\tfrac{1}{2}\omega t\right) \qquad (2)$$

With time-dependent coefficient, vectors $A$, and $B$ are new degrees of freedom vectors. Inserting Eq. (2) in Eq. (1), using trigonometric identities, factorization of sine and cosine terms and setting their coefficients equal to zero lead to a set of homogeneous equations. For a non-trivial solution with non-zero $A$ and $B$ vectors, the determinants of coefficient matrices should be set to zero and this leads to a so-called decoupled eigen-value problem as in Eq. (3).

$$\begin{pmatrix} K - a^S K_g^S + \tfrac{1}{2}a^D K_g^D & 0 \\ 0 & K - a^S K_g^S - \tfrac{1}{2}a^D K_g^D \end{pmatrix} - \frac{\omega^2}{4}\begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix} = 0 \qquad (3)$$

The solution of the above eigen-value problems for every assumed value of $(a^S, a^D)$ pair that leads to two instability boundary frequencies of the loading, surrounds the dynamic instability region

### 3- Results and Discussion

A cantilever eight layer laminated $[0/90]_{2S}$ plate of $L/b=10$ and $L/t=125$ is considered having through the width delamination at the middle thickness according to Fig. 2. The results of the first natural frequency, as well as fundamental buckling load, is calculated and compared with some references as shown in Table 1. The results show a very good accuracy of FSM calculations.

Dynamic Instability Regions (DIR) for a fully clamped square laminated plate [0/90/45/90]s under different static loading amplitudes are extracted and is depicted in Fig. 3. The plate is assumed to contain a central square delamination zone with the edge size of 30% of plate edges in the middle thickness. The results show that the effects of delamination are dependent on the amount of static component of the loading. In higher loading values, emerging delamination causes higher instability frequencies that mean somewhat stabilizing effect.
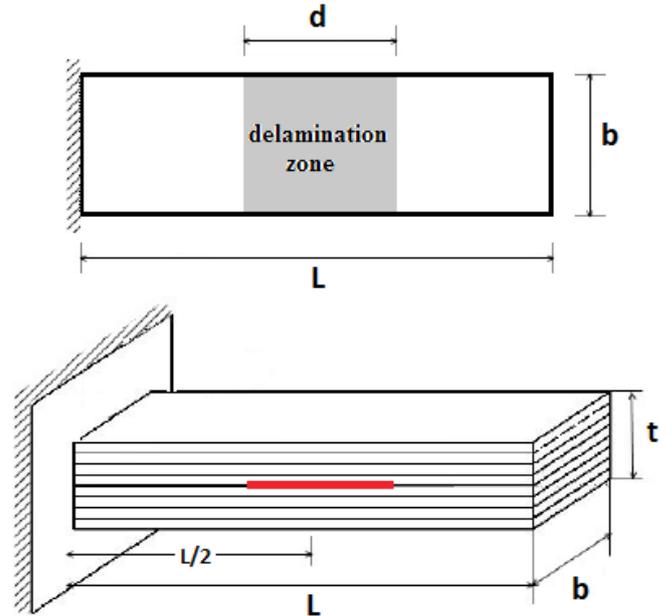


**Figure 2. Cantilever laminated plate with delamination region**

**Table 1. Fundamental buckling and natural frequency of cantilever plate with delamination**

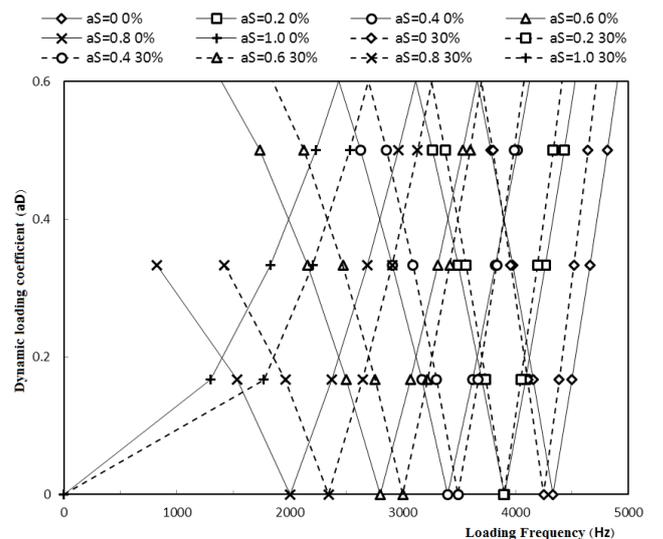| d/L | Experimental [1] | HST [2] | | HST FSM | |
|---|---|---|---|---|---|
| | $w_1$ | $w_1$ | $P_{cr}$ | $w_1$ | $P_{cr}$ |
| | [H] | [Hz] | [N] | [Hz] | [N] |
| 0 | 79.83 | 82.03 | 16.21 | 82.10 | 16.33 |
| 0.2 | 78.17 | 80.75 | 15.93 | 81.15 | 16.16 |
| 0.4 | 75.38 | 75.99 | 15.08 | 75.33 | 14.85 |
| 0.6 | 66.96 | 66.83 | 12.96 | 65.40 | 12.26 |



**Figure 3. Cantilever laminated plate with delamination region**

## 4- Conclusions

A B-spline version of finite strip method is formulated and applied to solve the parametric stability problem of laminated plates containing internal delaminations. A twin-layer modeling technique is utilized to take delamination effects into account. The loading is assumed to be of uniform stress field type with two components, namely static and time-varying parts. The analysis has been conducted in the context of a third-order shear deformation theory of shells and plates with the assumptions of free traction surfaces. Applications of the developed formulation are presented to investigate the capabilities and accuracy of the method. It is shown that FSM calculations are fairly accurate in analyzing such structures

## References

[1] M.-H.H. Shen, J.E. Grady, Free vibrations of delaminated beams, *AIAA Journal*, 30(5) (1992) 1361-1370.

[2] H.R. Ovesy, J. Fazilati, M.R. Mahmoudabadi, Finite strip buckling and free vibration analysis of laminated composite plates containing delamination using a first order layerwise theory, in: *Proceedings of the 12th international conference on computational structures technology*, Tehran, (2014).