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On the Free Vibration Analysis of a CNT-Reinforced Plate Bonded to a Magnetoelectroelastic Layer

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ABSTRACT: In this study, free vibration of a two-layered smart rectangular plate composed of a single-walled carbon nanotube-reinforced layer and a magnetoelectroelastic layer are investigated. Carbon nanotubes are distributed uniformly along the thickness of the composite layer. The temperature of the environment changes uniformly. The plate is simply-supported and subjected to electric and magnetic loadings. First-order shear deformation theory is used to determine the equations of motion of the plate, and Gauss's laws for electrostatics and magnetostatics are used to model the magnetoelectric behavior of the plate. By defining the generalized displacements of the plate in double Fourier series form and then by using orthogonality principle of trigonometric functions, the partial differential equations of motion are transformed into a set of algebraic equations in terms of the natural frequency of the plate. Therefore, an analytical relation is obtained for the fundamental natural frequency. After validation of the proposed model, some examples are presented to investigate the effects of several parameters on the free vibration response of this smart plate.

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1- Introduction

Carbon nanotubes have been the subject of many studies due to their excellent chemical and physical properties such as high relative stiffness and strength. Alibeigloo [1] investigated free vibration of a carbon nanotube-reinforced cylindrical shell with piezoelectric layers. Zhang et al. [2] investigated the vibration control of carbon nanotube-reinforced plates by using piezoelectric layers as sensor and actuator. The main property of magneto-electro-elastic (MEE) materials is the magneto-electric effect. Due to this effect, there exists a coupling between mechanical, electric, and magnetic fields. Pan [3] studied the static response of MEE rectangular plate for the first time. Shooshtari and Razavi [4] used third-order shear deformation theory (TSDT) to analyze the free vibration of MEE plates.

In this study, free vibration of a two-layered smart rectangular plate composed of a single-walled carbon nanotube-reinforced layer and an MEE layer are investigated based on the first-order shear deformation theory (FSDT) and Gauss's laws for electrostatics and magnetostatics.

2- Modeling the Problem

According to Fig. 1, the plate consists of two layers with *a* and *b* being its length and width, respectively. The thickness of the MEE layer is h_s and it is attached to the carbon nanotube-reinforced composite layer with *h* being its thickness. The nanotubes are distributed uniformly along the thickness of the composite layer.

The resultants of this hybrid plate are obtained by:

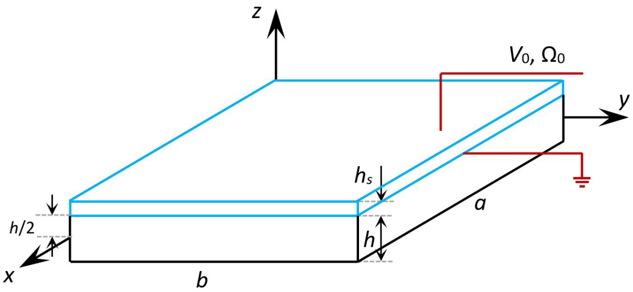

Figure 1. Schematic of the studied plate

$$
\begin{aligned}
&\{N_x\ N_y\ N_{xy}\}^T = \int_{-h/2}^{h/2}\{\sigma_x\ \sigma_y\ \sigma_{xy}\}_C^T\,dz + \\
&\int_{h/2}^{h/2+h_s}\{\sigma_x\ \sigma_y\ \sigma_{xy}\}_S^T\,dz\,, \\
&\{M_x\ M_y\ M_{xy}\}^T = \int_{-h/2}^{h/2}\{\sigma_x\ \sigma_y\ \sigma_{xy}\}_C^T\,zdz + \\
&\int_{h/2}^{h/2+h_s}\{\sigma_x\ \sigma_y\ \sigma_{xy}\}_S^T\,zdz\,, \\
&\{Q_x\ Q_y\}^T = K\int_{-h/2}^{h/2}\{\sigma_{xz}\ \sigma_{yz}\}_C^T\,dz + \\
&K\int_{h/2}^{h/2+h_s}\{\sigma_{xz}\ \sigma_{yz}\}_S^T\,dz
\end{aligned}
\tag{1}
$$

where subscripts *C* and *S* denote the carbon nanotube-reinforced composite layer and MEE layer, respectively, and *K* is the shear correction factor.

Constitutive relation of the carbon nanotube-reinforced composite layer is expressed, below:

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$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_{yz}\\ \sigma_{xz}\\ \sigma_{xy}\end{Bmatrix}_C = \begin{bmatrix}Q_{11} & Q_{12} & 0 & 0 & 0\\ Q_{12} & Q_{22} & 0 & 0 & 0\\ 0 & 0 & Q_{44} & 0 & 0\\ 0 & 0 & 0 & Q_{55} & 0\\ 0 & 0 & 0 & 0 & Q_{66}\end{bmatrix}_C \left(\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix} - \begin{Bmatrix}\alpha_{11}\\ \alpha_{22}\\ 0\\ 0\\ 0\end{Bmatrix}_C \Delta T\right) \qquad (2)$$

where the unknown parameters are defined in [5]. On the other hand, the constitutive relation of the MEE layer is expressed as follows:

$$\begin{aligned}\{\sigma\}_S &= [C]_S\cdot\left(\{\varepsilon\}-\{\alpha\}\Delta T\right)-[e]\{E\}-[q]\{H\}\\ \{D\} &= [e]^T\{\varepsilon\}+[\eta]\{E\}+[d]\{H\}+\{p\}\Delta T\\ \{B\} &= [q]^T\{\varepsilon\}+[d]\{E\}+[\mu]\{H\}+\{m\}\Delta T\end{aligned} \qquad (3)$$

where the vectors and matrices are defined in [6].

Thus, using Eqs. (1) to (3), equations of motion of the plate based on the FSDT [7] are obtained:

$$\alpha_1 u_{0,xx}+\alpha_2 u_{0,yy}+\alpha_3 v_{0,xy}+\alpha_4\theta_{x,xx}+\alpha_5\theta_{x,yy}+\alpha_6\theta_{y,xy}=0 \qquad (4)$$

$$\alpha_7 u_{0,xy}+\alpha_8 v_{0,xx}+\alpha_9 v_{0,yy}+\alpha_{10}\theta_{y,xx}+\alpha_{11}\theta_{y,yy}+\alpha_{12}\theta_{x,xy}=0 \qquad (5)$$

$$\alpha_{13} w_{0,xx}+\alpha_{14} w_{0,yy}+\alpha_{15}\theta_{x,x}+\alpha_{16}\theta_{y,y}=I_0 w_{0,tt} \qquad (6)$$

$$\begin{aligned}&\alpha_{17}u_{0,xx}+\alpha_{18}u_{0,yy}+\alpha_{19}v_{0,xy}+\alpha_{20}w_{0,xxx}+\\ &\quad\alpha_{21}w_{0,xyy}+\alpha_{22}w_{0,x}+\alpha_{23}\theta_{x,xx}\\ &\quad+\alpha_{24}\theta_{x,yy}+\alpha_{25}\theta_{y,xy}+\alpha_{26}\theta_x=0\end{aligned} \qquad (7)$$

$$\begin{aligned}&\alpha_{27}u_{0,xy}+\alpha_{28}v_{0,xx}+\alpha_{29}v_{0,yy}+\alpha_{30}w_{0,xxy}+\\ &\quad\alpha_{31}w_{0,yyy}+\alpha_{32}w_{0,y}+\alpha_{33}\theta_{x,xy}\\ &\quad+\alpha_{34}\theta_{y,xx}+\alpha_{35}\theta_{y,yy}+\alpha_{36}\theta_y=0\end{aligned} \qquad (8)$$

where the coefficients are constant functions of the plates properties which are not given here for brevity.

Now, by using the Navier method, one can obtain the following set of algebraic equations which gives the natural frequency of the plate:

$$\begin{bmatrix}L_1 & L_2 & 0 & L_3 & L_4\\ L_5 & L_6 & 0 & L_7 & L_8\\ 0 & 0 & \left(L_9-L_{10}\omega_0^2\right) & L_{11} & L_{12}\\ L_{13} & L_{14} & L_{15} & L_{16} & L_{17}\\ L_{18} & L_{19} & L_{20} & L_{21} & L_{22}\end{bmatrix}\begin{Bmatrix}U\\ V\\ W\\ X\\ Y\end{Bmatrix}=\mathbf{0} \qquad (9)$$

3- Results and Discussion

Tables 1 and 2 present the natural frequencies of the studied hybrid plate with $h = 1$ at $T = 300$ K for different electric and magnetic potentials. It is seen that the positive electric potential decreases the natural frequency, whereas the positive magnetic potential increases it. Moreover, *a/h* ratio has a significant effect on the natural frequency of the plate.

Table 1. Natural frequencies (rad/s) of the hybrid plate for different electric potentials ($h_s = 0.1h$, $V_{CNT} = 0.11$).

a/h	V_0 (V)		
	−1000	0	1000
10	258.2800	258.2795	258.2790
20	70.3172	70.3168	70.3163
50	11.5604	11.5599	11.5595

Table 2. Natural frequencies (rad/s) of the hybrid plate for different magnetic potentials ($h_s = 0.1h$, $V_{CNT} = 0.11$).

a/h	Ω_0 (V)		
	−1000	0	1000
10	258.2153	258.2795	258.3437
20	70.2578	70.3168	70.3757
50	11.5024	11.5599	11.6172

Fig. 2 shows the effects of aspect ratio (a/b) and volume fraction of the carbon nanotubes (VCNT) on the natural frequency of the plate. The effect of temperature on the natural frequency is also been studied and the result is shown in Table 3.

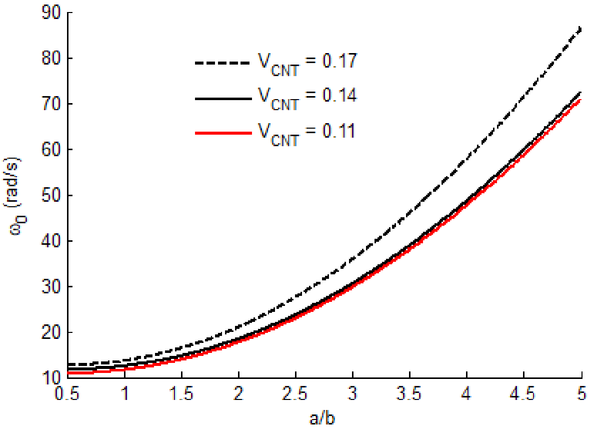

Figure 2. Effects of *a/b* and V_{CNT} on the natural frequency ($\Delta T = V_0 = \Omega_0 = 0$).

Table 3. Effect of temperature on the natural frequency of the hybrid square plate ($h_s = 2h = 2$ mm, $a/h = 20$).

V_{CNT}	*T* (K)		
	300	500	700
0.11	55.4084	47.3251	37.4027
0.14	56.4346	48.1520	37.9091
0.17	57.5405	48.9750	38.3264

It is observed from Fig. 2 that for the plate with $V_{CNT} = 0.17$, *a/b* ratio has more effect on the natural frequency of the plate. Table 3 shows that increasing the temperature, decreases the natural frequency of the plate.

4- Conclusions

In this paper, free vibration of a two-layered smart rectangular plate composed of a single-walled carbon nanotube-reinforced layer and an MEE layer are investigated based on the FSDT. Some examples are presented and it is found that (a) positive electric potential decreases the natural frequency, (b) for higher V_{CNT}, *a/b* ratio has more effect on the natural frequency, and (c) increasing the temperature, decreases the natural frequency.

References

[1] A. Alibeigloo, Free vibration analysis of functionally graded carbon nanotube-reinforced composite cylindrical panel embedded in piezoelectric layers by using theory of elasticity, *European Journal of Mechanics A/Solids*, 44(1) (2014) 104-115.

[2] L.W. Zhang, Z.G. Song, K.M. Liew, Optimal shape control of CNT reinforced functionally graded composite plates using piezoelectric patches, *Composites Part B: Engineering*, 85(1) (2016) 140-149.

[3] E. Pan, Exact solution for simply supported and multilayered magneto-electro-elastic plates, *Journal of Applied Mechanics*, 68(4) (2001) 608-618.

[4] A. Shooshtari, S. Razavi, Vibration analysis of a magnetoelectroelastic rectangular plate based on a higher-order shear deformation theory, *Latin American Journal of Solids and Structures*, 13(3) (2016) 554-572.

[5] P. Zhu, Z.X. Lei, K.M. Liew, Static and free vibration analyses of carbon nanotube-reinforced composite plates using finite element method with first order shear deformation plate theory, *Composite Structures*, 94(4) (2012) 1450-1460.

[6] Y. Ootao, Y. Tanigawa, Transient analysis of multilayered magneto-electro-thermoelastic strip due to nonuniform heat supply, *Composite Structures*, 68(4) (2005) 471-480.

[7] J.N. Reddy, *Mechanics of laminated composite plates and shells: theory and analysis*, 2nd Edition, CRC Press, 2004.

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