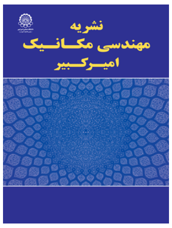

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Optimal laser control for cancer thermal therapy

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ABSTRACT: Todays, various treatments such as surgery, chemotherapy, radiotherapy, and hyperthermia are used to treat cancer. The best treatment for cancer is to accurately control the distribution of temperature in the damaged tissue, which has been the subject of many studies in recent years. Due to the increased temperature in cancer treatment, and especially in hyperthermia, the healthy tissue adjacent to the damaged tissue also disappears and results in bad consequences. In this paper, the optimal laser control for cancer therapy has been done. According to the non-Fourier behavior of temperature transitions in laser treatments, the time-dependent transient temperature distribution in one-dimensional mode, along with the heat of metabolism and perfusion of blood, using the Pence heat transfer equation, is analyzed. In order to minimize the damage to the healthy tissues adjacent to the damaged tissue, the objective function includes the difference between the calculated thermal damage with the desired thermal damage is defined. Therefore, the thermal flux value is optimized as an optimal control problem, and the lowest and most useful value is obtained. Finally, the results of the numerical solution to this problem are extracted and shown for triangular thermal flux and square heat pulses.

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1- Introduction

In hyperthermia, the temperature of the cancer tissue is usually increased to a certain temperature for a specified period of time. The performance of thermal therapy depends on the temperature and damage in the target tissue without damaging the surrounding healthy tissue. Pennes bioheat equation is the most widely applied model for temperature distribution in living biological tissues [1]. A variety of analytical and numerical techniques have been developed for the solutions of thermal behavior in biological tissues [2-4]. The aim of the present study is to develop a conjugate gradient method to optimize a heat source history that results in a desired thermal dose in a one-dimensional bioheat transfer process.

2- Controller Design

A one-dimensional tissue subjected to laser pulses is considered as schematically shown in Fig. 1.

The heat transfer process can be expressed by the Pennes bioheat transfer model as follows.

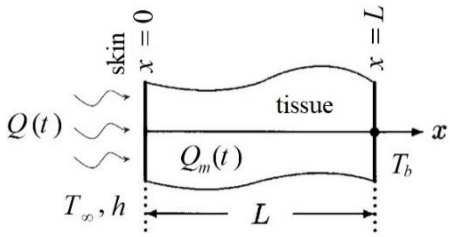

Fig. 1. Tissue subjected to laser pulses

$$
\begin{aligned}
&\rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} + W_b \left(T_b - T\right) + Q\left(t\right) + Q_m \\
&k \frac{\partial T\left(0,t\right)}{\partial x} = h\left[T\left(0,t\right) - T_\infty\right] \\
&T\left(L,t\right) = T_\infty \\
&T\left(x,0\right) = T_0
\end{aligned}
\tag{1}
$$

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The control problem is reduced to how to choose the function $Q(x,t)$ such that it minimizes the following objective function

$$J = \int_0^L \left[D(x) - D_d(x) \right]^2 dx + \int_0^L \int_0^{t_f} \left\{ \begin{array}{l} k \dfrac{\partial^2 T}{\partial x^2} + \rho_b c_b \omega_b \left(T_b - T \right) \\ + Q + Q_m - \rho c \dfrac{\partial T}{\partial t} \end{array} \right\} \varphi dt dx \tag{2}$$

$D(x)$ and $D_d(x)$ are, respectively, the computed and desired thermal dose over the thermal treatment period. For minimizing the function J under the constraints, the Lagrangian functional changed into an adjoint problem in the following form.

$$\begin{aligned} & -\rho c \frac{\partial \varphi}{\partial t} = k \frac{\partial^2 \varphi}{\partial x^2} - \rho_b c_b \omega_b \varphi + S \\ & k \frac{\partial \varphi(0,t)}{\partial x} = h \varphi(0,t) \\ & \varphi(L,t) = 0 \\ & \varphi(x,t_f) = 0 \end{aligned} \tag{3}$$

where

$$S = 2R^T \left(\int_0^{t_f} R^T dt - D_d \right) \left(T \frac{\partial R}{R \partial T} + LnR \right) \tag{4}$$

The variation ΔJ is derived after $q(t)$ is perturbed by $\Delta q(t)$ and $T(x, t)$ is perturbed by $\Delta T(x, t)$. Subtracting from the resulting expression the original and neglecting the second-order terms, the sensitivity problem is derived as follows.

$$\begin{aligned} & \rho c \frac{\partial \Delta T}{\partial t} = k \frac{\partial^2 \Delta T}{\partial x^2} + \rho_b c_b \omega_b \Delta T + \Delta Q(t) \\ & k \frac{\partial \Delta T(0,t)}{\partial x} = h \Delta T(0,t) \\ & \Delta T(L,t) = 0 \\ & \Delta T(0,t) = 0 \end{aligned} \tag{5}$$

The following iteration process based on the conjugate gradient method is used for the estimation of $q(t)$ by minimizing the above functional $J[q(t)]$.

$$Q^{(j+1)}(t) = Q^{(j)}(t) + \beta^{(j)} M^{(j)}, \quad \beta^{(0)} = 1 \tag{6}$$

Where parameters β, and M are calculated as follows.

$$\begin{aligned} & \beta^{(j)} = \frac{\int_0^L \int_0^{t_f} A\, B\, dt\, dx}{\int_0^L \left[\int_0^{t_f} C\, dt \right]^2 dx} \\ & M^{(j)} = -J'^{(j)} + \gamma^{(j)} M^{(j-1)} \end{aligned} \tag{7}$$

The parameters A, B, C, J' and γ in Eqs. (7) are also derived as follows.

$$\begin{aligned} & A = R^T dt - D_d(x) \\ & B = R^T \left(T \frac{1}{R} \frac{\partial R}{\partial T} - LnR \right) \Delta T \\ & C = \left(R^T \left(T \frac{1}{R} \frac{\partial R}{\partial T} - LnR \right) \Delta T \right) \\ & J'^{(j)} = -\varphi(0,t) \\ & \gamma^{(j)} = \frac{\int_0^{t_f} \left[J'^{(j)}(t) \right]^2 dt}{\int_0^{t_f} \left[J'^{(j-1)}(t) \right]^2 dx}, \quad \gamma^{(0)} = 0 \end{aligned} \tag{8}$$

The iterative process, Eq. (6), is repeated until each component of vector Q satisfies the following stopping criteria.

$$\left| \frac{Q^{(j+1)} - Q^{(j)}}{Q^{(j)}} \right| \le \varepsilon \tag{9}$$

3- Results and Discussion

The known square-wave pulse heat source used in generating the desired thermal dose $D_d(x)$ is plotted in Fig. 2. The variation of thermal dose in the iterative numerical process is shown in Fig. 3. The space and time temperature variation of the tissue was also depicted in Figs. 4 and 5, to show temperature field during the heating process.

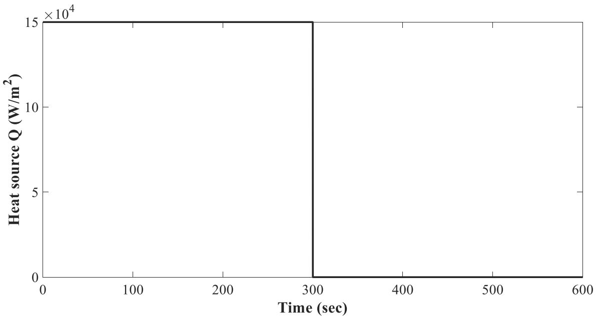

Fig. 2. Heat flux used for calculating desired thermal dose

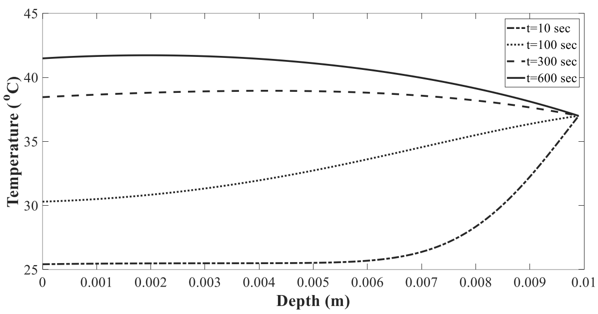

Fig. 4. The temperature of the tissue along the tissue at different times

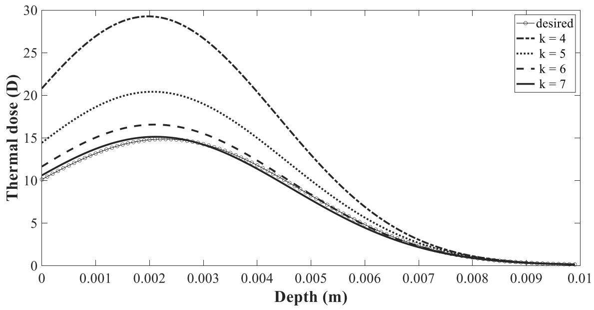

Fig. 3. Calculated thermal dosed in iterative numerical process

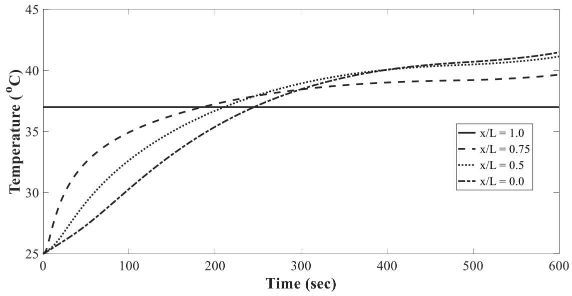

Fig. 5. The temperature of the tissue versus time in different locations

4- Conclusions

The conjugate gradient method was successfully applied for the solution of the hyperbolic heat conduction problem to determine the unknown time-dependent heat flux at the surface of living skin tissue while knowing the desired thermal dose in the tissue. Numerical results confirm that the proposed method can accurately estimate the optimal time-dependent surface heat flux for the problem to minimize the damage to the healthy tissues adjacent to the damaged tissue.

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