Investigating the nonlinear coupled vibrations of elastic blade of helicopter and analysis of flutter frequencies

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ABSTRACT

Aeroelastic instability in blades is one of the most important sources of instability in helicopter rotors, and the most critical of these instabilities is flutter. In this paper, in order to investigate the blade flutter and its relationship with the rotor structural parameters, using the Hamilton's principle and considering the Euler-Bernoulli beam theory, the coupled nonlinear partial differential equations governing the rotating elastic blade of a helicopter in the hover flight mode are extracted and converted into a set of ODEs by applying Galerkin method. Then the obtained equations for small perturbations are linearized around the steady state conditions. assuming the harmonic response, the natural frequencies of the blade in three motion axes are calculated and the relationship between the natural frequency and flutter frequency of the blade with structural and aerodynamic parameters are shown. Using numerical simulation, the results for two types of soft and stiff blades with given characteristics in terms of different parameters such as blade twist angle, pre-cone angle and rotation speed of rotor for the first mode shape are extracted. Finally, the effect of each of the mentioned parameters on the flutter frequency and also, the blade stability region is analyzed. It is shown that by increasing the blade stiffness, the flutter frequency will increase and the system will be stable.

KEYWORDS

Elastic Blade, Flutter, helicopter rotor, Aeroelastic, Instability

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1. Introduction

The flutter phenomenon is the most dangerous type of instability in the helicopter rotor and occurs in various reasons. Hodges and Ormiston [1] investigated the stability of elastic flap bending, lead-lag bending, and torsion of uniform and untwisted cantilever rotor blades with variable structural coupling in hover flight mode. Pardo et al. [2] have presented the development mathematical modeling and analysis of the natural frequencies and mode shapes of coupled flap-lag-torsion non-uniform rotor blades based on the Lagrange equations of motion. Kaya and Ozdemir [3] analyzed flutter stability of a uniform hingeless rotor blade in hover with structural coupling and demonstrated the effects of pitch angle and blade rotation speed. analyzing the aeroelastic stability of the curved composite hingeless rotor blade in the hover flight mode has been investigated by Amoozgar and Shahverdi [4]. Sarker and Chakravarty [5] have investigated of the coupled, steady-state dynamic response of the hingeless helicopter rotor blade at forward flight.

In this paper, the coupled nonlinear partial differential equations of the rotating elastic blade of a helicopter in the hover flight mode are extracted using the Hamilton's principle and considering the blade as a Euler-Bernoulli beam and converted into a set of ODEs by applying Galerkin method. the obtained equations are linearized around the steady state conditions for small perturbations and the flutter frequencies of the blade are analyzed.

2. Equations of motion

The coordinate system of an untwisted blade before and after deflections is considered as follows:

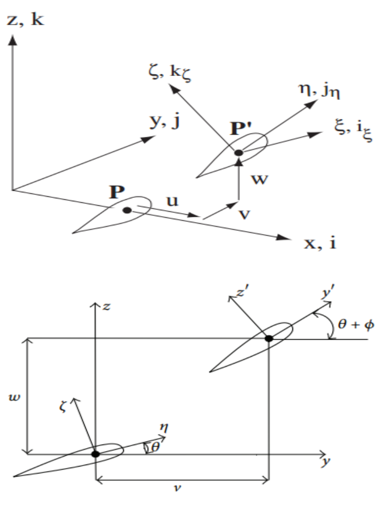


Figure 1. Coordinate system of rotating helicopter blade cross section before and after deformation [1, 6]

by applying the Galerkin method to the nonlinear variable coefficient equations and linearizing them around small perturbations, the governing equations of the blade in the dimensionless form are as follows [1]:

(1)

$$\begin{aligned}\delta v_i : \sum_{j=1}^{N}\Bigg\{ & v_j\left\{D_{ij}+\left[\Lambda_2-(\Lambda_2-\Lambda_1)\sin^2(\theta)\right]\beta_j^4\delta_{ij}-\delta_{ij}\right\}+w_j\frac{\sin(2\theta)}{2}(\Lambda_2-\Lambda_1)\beta_j^4\delta_{ij} \\ & +(\Lambda_2-\Lambda_1)\sum_{k=1}^{N}K_{jki}\phi_j\left[-v_k\sin(2\theta)+w_k\cos(2\theta)\right] \\ & -2\beta_{pc}\delta_{ij}\dot{w}_j+2\sum_{k=1}^{N}(F_{ikj}-F_{jki})v_k\dot{v}_j-2\sum_{k=1}^{N}F_{jki}w_k\dot{w}_j \\ & +\delta_{ij}\ddot{v}_j+\frac{\gamma}{6}\begin{bmatrix}\bar{v}_i I_{ij}\phi_j+(2\frac{c_{d_0}}{a}E_{ij}+\theta\bar{v}_i\delta_{ij}+\bar{v}_i\sum_{k=1}^{N}\phi_k H_{ijk})\dot{v}_j \\ +(2\bar{v}_i\delta_{ij}-\theta E_{ij}-\sum_{k=1}^{N}\phi_k G_{ijk})\dot{w}_j\end{bmatrix}\Bigg\} \\ & =\frac{\gamma}{6}(\bar{v}_i^2 A_i-\frac{c_{d_0}}{a}C_i+\bar{v}_i\theta B_i)\end{aligned}$$

(2)

$$\begin{aligned}\delta w_i : \sum_{j=1}^{N}\Bigg\{ & w_j\left\{D_{ij}+\left[\Lambda_1+(\Lambda_2-\Lambda_1)\sin^2(\theta)\right]\beta_j^4\delta_{ij}\right\}+v_j\frac{\sin(2\theta)}{2}(\Lambda_2-\Lambda_1)\beta_j^4\delta_{ij} \\ & +(\Lambda_2-\Lambda_1)\sum_{k=1}^{N}K_{jki}\phi_j\left[v_k\cos(2\theta)+w_k\sin(2\theta)\right]+2\beta_{pc}\delta_{ij}\dot{v}_j \\ & +2\sum_{k=1}^{N}F_{ikj}w_k\dot{v}_j+\ddot{w}_j(1+\frac{\gamma\bar{c}}{24})\delta_{ij} \\ & +\frac{\gamma}{6}\begin{bmatrix}-J_{ij}\phi_j+\sum_{k=1}^{N}L_{ijk}v_j w_j+\beta_{pc}E_{ij}v_j-\frac{\bar{c}}{2}O_{ij}w_j \\ -(2\theta E_{ij}+2\sum_{k=1}^{N}\phi_k G_{ijk}-\bar{v}_i\delta_{ij})\dot{v}_j+E_{ij}\dot{w}_j-\frac{3\bar{c}}{4}I_{ij}\dot{\phi}_j\end{bmatrix}\Bigg\} \\ & =-\beta_{pc}B_i+\frac{\gamma}{6}(-\bar{v}_i B_i+\theta C_i+\frac{\bar{c}}{2}\beta_{pc}B_i)\end{aligned}$$

(3)

$$\begin{aligned}\delta\phi_i : \sum_{j=1}^{N}\Bigg\{ & \phi_j\left\{\mu^2 KN_{ij}+\left[k\gamma_j^2+(\mu_2^2-\mu_1^2)\cos(2\theta)\right]\delta_{ij}\right\} \\ & +(\Lambda_2-\Lambda_1)\sum_{k=1}^{N}K_{ijk}\left[\frac{\sin(2\theta)}{2}(w_j w_k-v_j v_k)+\cos(2\theta)v_j w_k\right] \\ & +\mu^2\delta_{ij}\ddot{\phi}_j+\frac{\gamma\bar{c}^2}{48}M_{ij}\dot{\phi}\Bigg\} \\ & =-(\mu_2^2-\mu_1^2)\frac{\sin(2\theta)}{\sqrt{2}\gamma_i}\end{aligned}$$

In order to investigating of the flutter, the response is assumed to be harmonic and so from equations (1) to (3) it can be written as follows:

$$\left|-\omega^2[M]+i\,\omega[D]+[K]\right|=0 \qquad (4)$$

M, D, K are mass, damping and stiffness matrices.

3. Results and Discussion

flutter determinant variations with frequency and dimensionless flutter frequency $)\omega_f($ for a blade with given parameters [1] are shown in "figure 2".

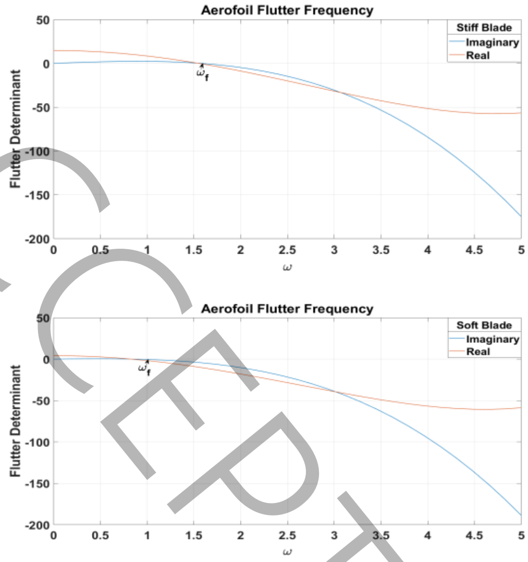


Figure 2. flutter determinant Vs. frequency for

$\Omega = 100 \, {}^{rad}\!/_{s}$ **and** $\theta = 8^{\circ}, \beta_{pc} = 5^{\circ}$

As shown in “Figure 2”, for a stiff blade, the flutter frequency (intersection of imaginary part with the zero axis) is about 1.57 and since the imaginary part intersects the zero axis in the negative region of the real part, the system will stable and the stability region is placed after the frequency point of 1.55 and also, for the flexible blade, the flutter frequency is about 0.82 and the stability region is placed after 0.826. it can be concluded that with increasing of the blade stiffness, the flutter occurred later and the system will be stable.

the flutter determinant variations with blade rotation speed, pitch angle θ and precone angle β_{pc} are shown in “Figure 3” and “Figure 4”.

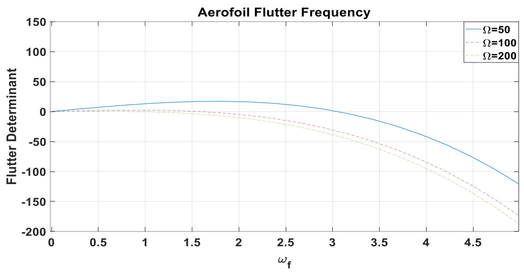
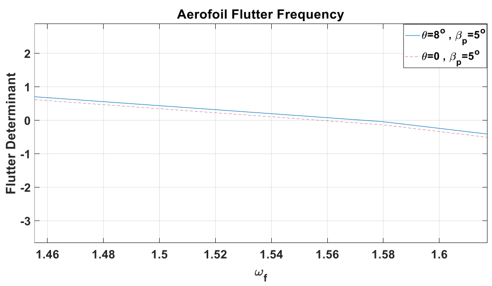


Figure 3. flutter frequency variations with rotor speed for

$\theta = 8^{\circ}, \beta_{pc} = 5^{\circ}$



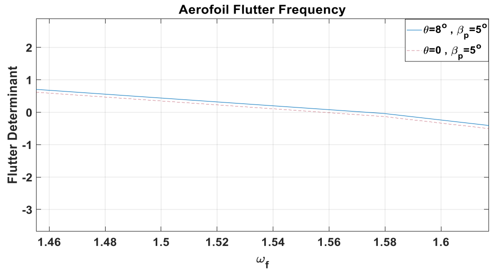


Figure 4. flutter frequency variations for pitch angle θ and precone angle β_{pc}

As it can be seen from “Figure 3,4” with increasing of the rotation speed, the flutter frequency decreases. as well as with increasing of the pitch angle, the flutter frequency will increases and increasing the pre-cone angle decreses the flutter frequency slightly.

4. **Conclusions**

1- By increasing the stiffness of the blade, the flutter frequency increases and the flutter occurs later and since the flutter frequency is placed in the negative region of the real values, the system will be stable.
2- By increasing the rotation speed of the rotor, while the flutter frequency decreases, the system is stable.
3- As the pre-cone angle increases, the flutter frequency is slightly reduced and with increasing of pitch angle, the flutter frequency increases.

5. Reference

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