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Linear and Nonlinear Free Vibration of a Functionally Graded Magneto-electro-elastic Rectangular Plate Based on the Third Order Shear Deformation Theory

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ABSTRACT: In this paper, linear and nonlinear free vibration of one functionally graded magneto-electro-elastic rectangular plate is studied. The boundary conditions in all side of plate have been considered as simply supported. Also the equations of motions have been derived calculation the kinetic energy and potential energy based on the third order shear deformation theory using Hamilton principle. Considering the top surface of the plate as an pizeomagnetic material and the bottom surface as a piezoelectric material, the bottom and upper surfaces of the plate are subjected to electric and magnetic potentials. The electric and magnetic behaviors of the plate are modeled by using Gauss's laws. Then, the equations of motions have been transformed from partial differential equations to ordinary differential equations by using Galerkin Method. Then, Using Lindeshtot- Poincare method a closed form expression for linear and nonlinear natural frequency has been obtained. for validation of the proposed model, some numerical examples have been presented and comparisons between the obtained results with the results in literature have been down. It is shown that good agreement exist between obtained results and previous works. Then, to study the effects of several parameters on the nonlinear vibration response of functionally graded magneto-electro-elastic rectangular plates.

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1- Introduction

In recent years, magneto-electro-elastic materials have been the topic of many researches due to their ability to convert electrical, magnetic, and mechanical energy forms to each other. Pan [1] studied the response of a laminated magneto-electro-elastic plate analytically for the first time. Li and Zhang [2] used Mindlin's theory to determine the natural frequencies of a magneto-electro-elastic plate resting on an elastic foundation. Xue et al. [3] analyzed the large deflection of a magneto-electro-elastic thin plate based on the classical plate theory. Shooshtari and Razavi [4] investigate the linear and nonlinear free vibrations of laminated magneto-electro-elastic plates based on the first order shear deformation theory. In this paper, effects of several parameters on the nonlinear free vibration of a functionally graded magneto-electro-elastic plate is investigated based on the Third Order Shear Deformation (TSDT) plate theory in conjunction with single-mode Galerkin and Lindeshtod- Poincare method.

2- Modelling the Problem

Constitutive equations of a magneto-electro-elastic material are expressed by [4]:

$$\mathbf{C}=\begin{bmatrix} C_{11}(z) & C_{12}(z) & 0 & 0 & 0 \\ C_{21}(z) & C_{22}(z) & 0 & 0 & 0 \\ 0 & 0 & C_{44}(z) & 0 & 0 \\ 0 & 0 & 0 & C_{55}(z) & 0 \\ 0 & 0 & 0 & 0 & C_{66}(z) \end{bmatrix} \tag{1}$$

$$\mathbf{e}=\begin{bmatrix} 0 & 0 & e_{31}(z) \\ 0 & 0 & e_{32}(z) \\ 0 & e_{24}(z) & 0 \\ e_{15}(z) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, q=\begin{bmatrix} 0 & 0 & q_{31}(z) \\ 0 & 0 & q_{32}(z) \\ 0 & q_{24}(z) & 0 \\ q_{15}(z) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{\epsilon}=\begin{bmatrix} \epsilon_{11}(z) & 0 & 0 \\ 0 & \epsilon_{22}(z) & 0 \\ 0 & 0 & \epsilon_{33}(z) \end{bmatrix}, \boldsymbol{\mu}=\begin{bmatrix} \mu_{11}(z) & 0 & 0 \\ 0 & \mu_{22}(z) & 0 \\ 0 & 0 & \mu_{33}(z) \end{bmatrix}$$
$$\mathbf{d}=\begin{bmatrix} d_{11}(z) & 0 & 0 \\ 0 & d_{22}(z) & 0 \\ 0 & 0 & d_{33}(z) \end{bmatrix} \tag{2}$$

where C_{ij}, e_{31}, q_{31}, η_{33}, d_{33}, and μ_{33} are stiffness coefficient, piezoelectric, piezomagnetic, dielectric, magneto-electric, and magnetic permeability constants, respectively. σ_i, ϕ, D_z, ψ, and B_z denote stress, electric potential, electric displacement along z-axis, magnetic potential, and magnetic flux density along z-axis, respectively.

The plate is $CoFe_2O_4$-rich at $z=+h/2$ and $BaTiO_3$-rich at $z=-h/2$, and the material vary along the z-axis. Volume fraction of the piezoelectric phase (i.e., $BaTiO_3$) based on the power law is determined by:

$$V_B=\left(\frac{2z+h}{2h}\right)^p \tag{3}$$

where p is a non-negative real number and B denotes the piezoelectric phase. Then, material properties of the plate can

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be obtained by the following equation:

$$
\begin{aligned}
C_{ij}(z) &= (C_B - C_C)\left(\frac{2z+h}{2h}\right)^p + C_C \\
e_{ij}(z) &= (e_B - e_C)\left(\frac{2z+h}{2h}\right)^p + e_C \\
q_{ij}(z) &= (q_B - q_C)\left(\frac{2z+h}{2h}\right)^p + q_C \\
\in_{ij}(z) &= (\in_B - \in_C)\left(\frac{2z+h}{2h}\right)^p + \in_C \\
\mu_{ij}(z) &= (\mu_B - \mu_C)\left(\frac{2z+h}{2h}\right)^p + \mu_C
\end{aligned} \tag{4}
$$

Equations of motion of a Functionally Graded Magneto-electro Elastic Plate (FGMME) based on the third order shear deformation theory are expressed by:

$$
\begin{aligned}
& N_{xx,x} + N_{xy,y} = I_0 u_{,tt} + (I_1 - c_1 I_3)\phi_{x,tt} - c_1 I_3 w_{0,tt} \\
& N_{yy,y} + N_{xy,x} = I_0 v_{0,tt} + (I_1 - c_1 I_3)\phi_{y,tt} - c_1 I_3 w_{0,tt} \\
& \left(N_{yy,y} + N_{xy,x}\right) w_{0,y} + \left(N_{xx,x} + N_{xy,y}\right) w_{0,x} + N_{yy} w_{0,yy} \\
& + N_{xy} w_{0,xy} + N_{xx} w_{0,xx} + c_1\left(P_{xx,xx} + 2P_{xy,xy} + P_{yy,yy}\right) \\
& + \left(Q_{x,x} - c_2 R_{x,x}\right) + \left(Q_{y,y} - c_2 R_{y,y}\right) \\
& = I_0 w_{0,tt} - c_1^2 I_6\left(w_{0,ttxx} + w_{0,ttyy}\right) + c_1 I_3\left(u_{0,tt} + v_{0,tt}\right) \\
& \quad + c_1\left(I_4 - c_1 I_6\right)\left(\phi_{x,xtt} + \phi_{y,ytt}\right) - c_1^2 I_6\left(w_{0,xxtt} + w_{0,yytt}\right) \\
& M_{xx,x} + M_{xy,y} - c_1 P_{xx,x} - c_1 P_{xy,y} + \left(Q_x - c_2 R_x\right) \\
& = -c_1\left(I_4 - c_1 I_6\right) w_{0,ttx} + \left(I_2 - 2c_1 I_4 + c_1^2 I_6\right)\phi_{x,tt} \\
& \quad + \left(I_1 - c_1 I_3\right) u_{0,tt} - c_1\left(I_4 - c_1 I_6\right) w_{0,xtt} \\
& M_{yy,y} + M_{xy,x} - c_1 P_{yy,y} - c_1 P_{xy,x} + \left(Q_y - c_2 R_y\right) \\
& = -c_1\left(I_4 - c_1 I_6\right) w_{0,ytt} + \left(I_2 - 2c_1 I_4 + c_1^2 I_6\right)\phi_{y,tt} \\
& \quad + \left(I_1 - c_1 I_3\right) v_{0,tt} - c_1\left(I_4 - c_1 I_6\right) w_{0,ytt}
\end{aligned} \tag{5}
$$

Assuming the simply support boundary condition in all edge of the plate and using the following shape functions which satisfy the boundary condition:

$$
\begin{aligned}
u_0(x,y,t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} U_{mn}(t)\,\mathrm{Cos}\,\alpha x\ \mathrm{Sin}\,\beta y \\
v_0(x,y,t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} V_{mn}(t)\,\mathrm{Sin}\,\alpha x\ \mathrm{Cos}\,\beta y \\
w_0(x,y,t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} hW_{mn}(t)\,\mathrm{Sin}\,\alpha x\ \mathrm{Sin}\,\beta y \\
\phi_x(x,y,t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} X_{mn}(t)\,\mathrm{Cos}\,\alpha x\ \mathrm{Sin}\,\beta y \\
\phi_y(x,y,t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Y_{mn}(t)\,\mathrm{Sin}\,\alpha x\ \mathrm{Cos}\,\beta y
\end{aligned} \tag{6}
$$

and then substituting in Eq. (5) and finally applying the Galerkin method on the resulting equations, one can obtain the following nonlinear differential equation:

$$
Z_1 W_{,tt} + Z_2 W + Z_3 W W_{,tt} + Z_4 W^2 + Z_5 W^3 = 0 \tag{7}
$$

which in non-dimensional form can be written as

$$
W_{,\tau\tau} + \omega^2 W + \alpha_1 W W_{,\tau\tau} + \alpha_2 W^2 + \alpha_3 W^3 = 0 \tag{8}
$$

It is seen that this equation includes quadratic and qubic nonlinearity terms. Following the procedure Lindeshtot-Poincare which is presented by Nayfeh and Mook [5], Eq. (8) is solved and the ratio of nonlinear frequency to linear frequency is obtained as:

$$
\frac{\omega_{NL}}{\omega_L} = \left[1 + \left(\frac{5}{6}\frac{\alpha_2}{\omega^2}\frac{\alpha_1\omega^2 - \alpha_2}{\omega^2} + \frac{3}{4}\frac{\alpha_3}{\omega^2} + \frac{\alpha_1}{12}\frac{\alpha_1\omega^2 - \alpha_2}{\omega^2}\right) r^2\right]^{1/2} \tag{9}
$$

3- Results and Discussion

To validate the proposed solution, the linear natural frequency of an MEE plate which is obtained from the present work has been compare with the literature and showed in Table 1.

Table 1. Linear natural frequency of an MEE square plate

Method	*(m,n)*		
	(1,1)	**(1,2)**	**(2,2)**
Moita et. al [6]	2.3965	4.5594	6.2256
Present	2.3997	4.6874	6.5002

In Fig. 1 one can see the effect of ratio of thickness to dimension of plate on the linear natural functionally graded magneto electro elastic rectangular plates.

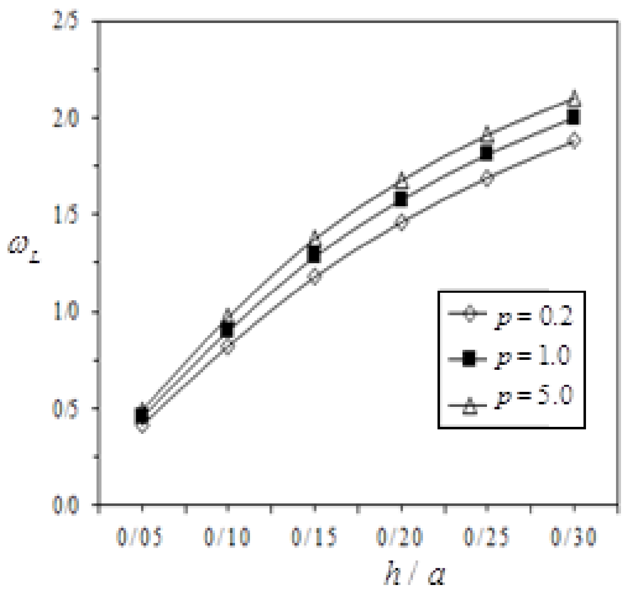

Figure 1. Effect of thickness to dimension of a MEE square

Also in Fig. 2 the effects of parameter of power law number in FGM model (p) on the ratio of nonlinear to linear natural frequency of plate has been shown.

Considering these two figures one can show that by increasing the number of power law, both linear and nonlinear natural frequencies will be increased.

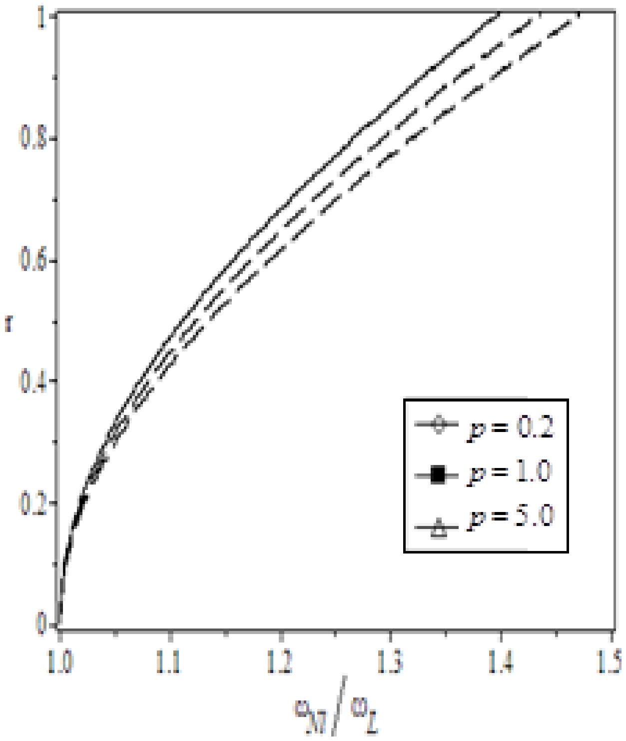

Figure 2. Effect of coefficient of power law number *p* on the ratio of nonlinear to linear natural frequency

4- Conclusions

In this paper, linear and nonlinear free vibrations of a functionally graded magneto-electro-elastic plate is investigated based on the third order shear deformation plate theory along with Lindeshtot-Poincare method. Several examples are given to validate the proposed solution and to investigate the effects of some parameters on the vibration response of this plate.

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