



# Damping Modeling in Dual Axis Torsion Micro-Actuators Considering the Bending of the Supporting Beams

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**ABSTRACT:** Torsional micro-actuators are employed in a variety of applications such as optical switches and biomedical imaging. Squeezed film damping is one of the important energy loss mechanisms in these systems. This kind of damping is a key factor in the performance characterization of micro-electro-mechanical systems and has been paid attention by many researchers. The objective of this paper is modeling the squeeze film air damping in dual axis torsional micro-actuators by considering the bending of the supporting torsion beams. To do so, first, the air inertial effects is neglected compared to its viscosity and the Reynolds equation governing the behavior of trapped air between the actuator and the underneath plate is simplified. The resulting equation is then normalized and solved using the extended Kantorovich method for obtaining the air pressure distribution under the plate. This pressure distribution is then employed for finding the damping force and torques. A parametric study is also carried out to determine the effect of different design parameters on the damping of the system. The results of this paper can be effectively employed for accurate dynamic modeling of dual axis torsional micro-actuators.

## 1- Introduction

Torsional micro-actuators have found a variety of applications in optical switches and biomedical imaging [1]. Squeezed film damping in these structures affects the performance of the device. So, many researchers have studied the effect of this damping on the behavior of micro structures. For example, Moeenfar et al. [2] used the Extended Kantorovich Method (EKM) to solve the squeezed film damping in single axis micromirrors. Malihi et al. [3] modeled the squeezed film damping investigated the effect of squeezed film damping on the stability of micro structures. Moeenfar and Ahmadian [4] studied the effect of bending of the supporting beams on the squeezed film damping of single axis torsional micro-actuators.

In previous studies, the problem of modeling squeeze film damping of micron sized dual axis torsional actuators have not been investigated. More specifically, the bending of the supporting torsion beams on the damping forces and torques had not been modeled yet. So in this study, the mentioned problem will be simulated and the relevant results are presented.

## 2- Methodology

### 2- 1- Mathematical modeling

Fig. 1 shows a schematic view of a dual axis torsional micro-actuator.

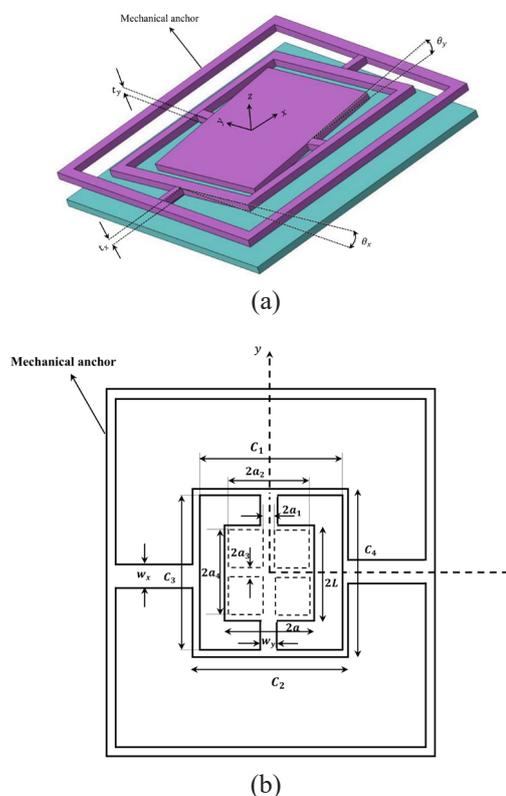


Fig. 1. Schematic (a) three and (b) two dimensional view of the torsional micro actuator.

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Neglecting the inertial effects and assuming incompressible conditions, the governing Reynolds equations become

$$\frac{\partial^2 \bar{p}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{p}}{\partial \bar{y}^2} = \frac{12\mu}{h^3} \frac{dh}{dt} \tag{1}$$

where  $\bar{p}$  is the underneath pressure and  $h$  is the gap between the electrode and the actuator and can be easily derived as  $h = h_0 - \delta_x - \delta_y + \bar{y}\theta_x + \bar{x}\theta_y$ . By substituting  $h(t)$  into Eq. (1), one can get

$$\frac{\partial^2 P}{\partial x^2} + \Gamma^2 \frac{\partial^2 P}{\partial y^2} = -\frac{x\eta_1}{\eta_2} + y - \frac{\eta_3}{\eta_2} \tag{2}$$

where  $\Gamma$  is the aspect ratio of the actuator plate. The normalized boundary conditions are also derived as

$$p(\pm 1, y) = p(x, \pm 1) = 0 \tag{3}$$

**2-2- Solution of the damping equation using extended Kantorovich method**

For solving Eq. (2), three new variables  $P_1$ ,  $P_2$  and  $P_3$  are introduced. The variables shall satisfy the following sets of equations and boundary conditions.

$$\frac{\partial^2 P_1}{\partial x^2} + \Gamma^2 \frac{\partial^2 P_1}{\partial y^2} = -x \tag{4}$$

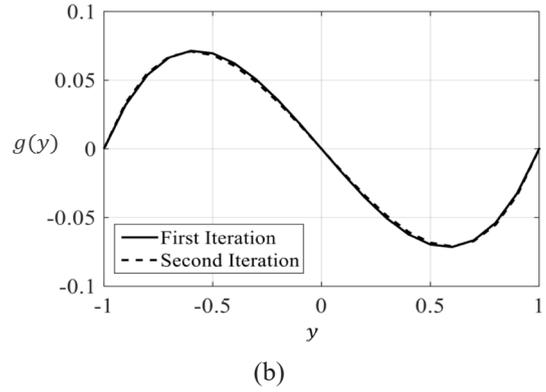
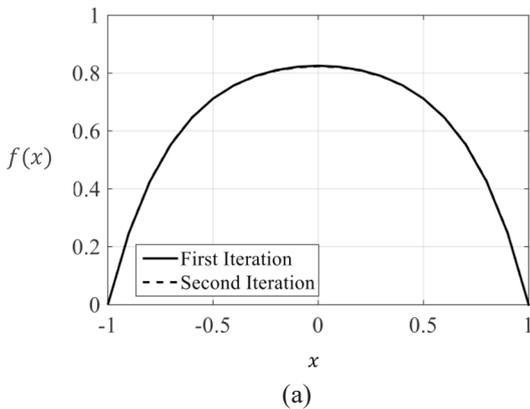
$$\frac{\partial^2 P_2}{\partial x^2} + \Gamma^2 \frac{\partial^2 P_2}{\partial y^2} = -1 \tag{5}$$

$$\frac{\partial^2 P_3}{\partial x^2} + \Gamma^2 \frac{\partial^2 P_3}{\partial y^2} = y \tag{6}$$

$$\begin{cases} P_i(\pm 1, y, t) = 0 \\ P_i(x, \pm 1, t) = 0 \end{cases}, i = 1, 2, 3 \tag{7}$$

**3- Results and Discussion**

Using the EKM, these equations can be solved as the multiplication of  $f(x)$  and  $g(y)$  both of which are derived in the solution process. The EKM provides very fast convergence. Fig. 2 shows that even a single iteration provides very satisfactory results.



**Fig. 2. Functions (a)  $f(x)$  and (b)  $g(y)$  at different iterations.**

The damping force and torques can also be derived by integrating the pressure obtained from EKM as

$$F_d(t) = \frac{\bar{F}_d(t)}{aL\eta_3} = \frac{\eta_2}{\eta_3} \iint_R P(x, y, t) dx dy \tag{8}$$

$$R = [-1, 1] \times [-1, 1]$$

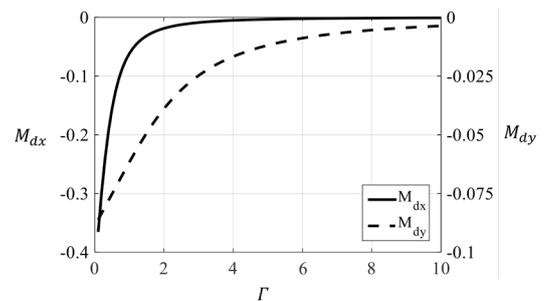
$$M_{dx}(t) = \frac{\bar{M}_{dx}(t)}{aL^2\eta_2} = \iint_R y P(x, y, t) dx dy \tag{9}$$

$$R = [-1, 1] \times [-1, 1]$$

$$M_{dy}(t) = \frac{\bar{M}_{dy}(t)}{La^2\eta_1} = \frac{\eta_2}{\eta_1} \iint_R -x P(x, y, t) dx dy \tag{10}$$

$$R = [-1, 1] \times [-1, 1]$$

In Fig. 3 the variation of the normalized damping torques along both  $x$  and  $y$  axes are depicted. It is observed that with increasing the aspect ratio of the plate, damping torques in both directions are increased.



**Fig. 3. Normalized squeeze film damping torques  $M_{dx}$  and  $M_{dy}$  vs  $\Gamma$**

**4- Conclusion**

The technology of optical microelectromechanical systems is growing rapidly. Torsional micro actuators are one of the most important optical Micro Electro Mechanical System (MEMS) devices. They work based on the capacitive micro plates which are moving towards each other and consequently, the trapped air in between can produce an energy loss mechanism known as squeezed film damping, which affects the performance of torsional micro actuators. So the objective of this paper was modeling and simulation of this squeezed film damping. On the other hand, the bending of the supporting torsion beams of

the actuator significantly affects the damping force and torques of the squeezed film damping problem. So in this paper, this effect was modeled. The solution procedure was carried out using EKM which was shown to have a very remarkable convergence speed. The simulation results revealed that the deflection of the beams is very important on the damping of the system and its ignorance may lead to appreciable accuracy loss in predicting the structure behavior. They can be accurately employed for simulation of the nonlinear dynamic behavior of dual axis torsional micro actuators with considering the bending effects.

## 5- References

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