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# Single and Multi-objective Optimal Control Design by Genetic Programming and Comparison with Riccati Equation Solutions

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**ABSTRACT:** Gaining the function of control signal that transfer the system states from initial to desired final conditions is one of the main issues related to the optimal control of modern systems. Optimal control signal is usually obtained by numerical solution (such as dynamic programming algorithm) or analytical solution (like Hamilton-Jacobi-Bellman or Riccati equations approaches) of a single-objective performance index which is a weighted combination of control effort and the fitness of system's states. However, choosing proper weight coefficients in these approaches needs a lot of trial and error in addition to experience. In this papers, such time consuming procedures are eliminated by using Genetic programming in single and multi-objective optimization process to find those closed-form mathematical solutions of optimal control problems. In this way, it would be readily possible to trade-off among the objective functions using the obtained pareto-front of those solutions based on the needs of the control system designer. It will be shown that in the case of same weighting factors, the solution of the Riccati equation would also be obtained using the approach of this paper.

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## 1- Introduction

The main aim in the subject of optimal control is finding a control signal which leads to convergence of the states toward the desired ones in the best way. There are many controls which can do such switching in the states of a dynamic system; however, only a few of them do it optimally. These control signals are commonly obtained by solving a performance index, which is a combination of several objectives with suitable weighting coefficients. There are a few approaches to find such optimal controls [1-3].

Rounding error in recurrence correlations, long-lasting time for loops and considerably high inaccuracy of solutions in addition to the single-objectivity of the ordinary approaches made researchers seek another approach, like a direct algorithm, for solving the optimization problems [4]. The Genetic Programming (GP) is an invaluable member of the evolutionary algorithms, which is a branch of the direct algorithm, whose efficiency in obtaining analytical solutions has been proved in many previous studies [5,6].

Some of the main deficiencies of the single-objective analysis could be highlighted as follow: 1) limitation of the designer's perspective on specifying the scalar values of the weighting coefficients, 2) Difficulty in choosing proper approaches to solve a single-objective problem and 3) Sensitivity of the final solution to the selected weighted coefficients.

In this work, a new modified GP algorithm is introduced whose efficacy is validated through the comparison of the single-objective solution with the exact Riccati solutions in a specific case study. Thereafter, the multi-objective analysis for the optimal control problem is performed by omitting the weighting coefficients, which is then appraised by evaluating the optimal values of the separated cost functions on the Pareto fronts. Using the proposed algorithm and the multi-objective analysis, many non-dominant points are provided for the designer to choose any desired one according to the design requirements. In this way, the viewpoint of the design and analysis in the field of optimal control would be expanded.

## 2- The Suggested Genetic Programming Algorithm

Genetic programming algorithm is an evolutionary method whose individuals are typical mathematical structures which are considered as solutions of a problem. The individual in this algorithm is shown in a tree structure in which variables and operators are placed in leaves and interior nodes of the tree, respectively. In GP, the initial population including a set of mathematical solutions is created randomly and then, in the next generation, premier individuals are preserved or created through an evolutionary procedure.

In this evolutionary trend, not only do individuals with more optimal values have more chance to be preserved, but also they are selected for producing new individuals of the following generation through the specific generation operators. After several cycles of the GP algorithm, it eventually leads to a generation with a set of optimal solutions. However, in the ordinary GP algorithm, the information of subtrees is totally overlooked, which finally causes the generated population to be less efficient.

In this study, the GP algorithm is promoted in a way that the subtree information is paid more attention, which makes the

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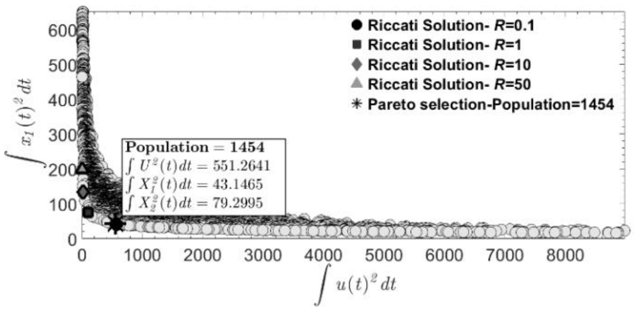


**Fig. 1. Pareto front of the aircraft problem in the control effort and the first state trajectory viewpoint**

population of the following generations more efficient than the previous version of GP. Besides, there are specific configurations that cause the proposed GP algorithm to be perfectly suited to either single or multi objective optimization problems.

## 3- Optimizing Using the Suggested GP Algorithm

### 3- 1- The single-objective optimization

The main goal in the optimal control subject is to find a control signal which causes the system states to get to the final desired conditions and, at the same time, minimizes the quadratic performance index defined as follows:

$$J_Q^*(X(t),U_Q^*(t)) = \int_0^\infty ([X(t)]_{1\times n}^T Q(t)_{n\times n}[X(t)]_{n\times 1} + [U^*(t)]_{1\times 1}^T R(t)_{1\times 1}[U_Q^*(t)]_{1\times 1})dt. \quad (1)$$

A particular case study where the control of the aircraft path was investigated has been selected for the validation of the suggested algorithm. In the previous study, the final solutions of this optimal problem were shown to be too sensitive to the selected weighting factors [7]. The steady space equation and the quadratic performance index J are respectively demonstrated as follow:

$$\dot{x}_1(t) = x_2(t),\ \dot{x}_2(t) = u(t),\quad \left(x(0)=10, x(0)=0\right). \quad (2)$$

$$J = \int^\infty [4\cdot x_1^2(t) + Ru^2(t)]dt,\quad \left(R = 0.1,1,10,50\right). \quad (3)$$

The exact optimal controls for such problems, which are in the specific form of linear feedback called control law, have been obtained through solving an equation typically known as Riccati equation. For this optimal control problem, the suggested GP in this paper has been configured with 200 populations, the terminal set of T={X1, X2} and the function of F= {Plus, Times, Minus}. Based on the comparisons between the solutions, the proposed GP solutions successfully converged to those obtained from the Riccati equation. Furthermore, all final suggested GP solutions are appropriately represented in the control law form while the error percentage of the worst solution is 0.003%. But with the same GP configuration, not only can the ordinary GP not represent the solutions in the control law form, but also the convergence trends of the ordinary GP solutions are worse than the suggested one in this paper.

### 3- 2- The multi-objective optimization

In this section, all weighting coefficients (Q and R) would be omitted. Thus, unlike the single-objective analysis of the previous section where one objective was defined, the three cost functions of control effort, first and second state trajectories are defined and separately evaluated. The suggested GP would be configured the same as the previous section with the difference that the new population build method, specific to multi-objective analysis, is used. It is worth noting that a container is considered that was heaped with the populations of the last four generations and the non-dominant points for the mentioned problem would be obtained by comparison of all individuals in this container with each other.

The 10565 non-dominant points are acquired by the proposed GP. Since there were three objectives in the specified problem, the non-dominant can be demonstrated in a 3 dimensional Pareto front. The obtained non-dominant points were evaluated in the perspective of the control effort and the first state trajectory through which the single-objective problem was investigated. In this viewpoint, the single objective points were expected to truly be within the boundary of the optimal non-dominant points. For further assessment, the nearest non-dominant point to the optimal single-objective point with the weighting factor value of R=0.1 has been selected. The specifications of all non-dominant points, the selected point and the optimal single-objective solutions have been illustrated in Fig. 1.

Although the optimum value of the selected non-dominant point is shown to be insignificantly worse than the same value of the single objective point, its optimality in the objective functions of the control effort and the second state trajectory is quite remarkable. In addition to the selected sample, there are many optimal non-dominant points which could be selected according to the design requirements.

## 4- Conclusions

In this work, the functions of the optimal control were obtained using a developed kind of genetic programming algorithm. Since this algorithm doesn't have the limitation of the ordinary approaches in the optimal control to evaluate the analytical solutions, the function of optimal control in the single and multi-objective analysis is obtained through an evolutionary subsequence without using any complex mathematical computations. The objectives which are studied in this paper were the control effort and the trajectory of the states.

In terms of generating new populations, GP was upgraded in a way that, in the single objective problems, the algorithm convergence towards the final solution is faster and more accurate than the ordinary GP. Also, its diversity was promoted to produce more non-dominant points for the multi-objective analysis. In addition, with the comparison of the suggested GP solutions with those obtained from the ordinary algorithm, the claim about the development of the suggested GP is eventually proved.

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