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Uncertainty Propagation Analysis in Free Vibration of Uncertain Composite Plate Using Stochastic Finite Element Method

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ABSTRACT: Material uncertainty is more widespread in composite material than the other engineering materials. This uncertainty makes response of these types of structures to be nondeterministic. In order to predict structural reliability, uncertainty in structural responses must be quantified. There is not a reported research in the literature studying free vibration of composite plate with spatially stochastic material properties. In this research, physical and mechanical properties of composite plate including tensile and shear modulus and density of the plate are modeled as stochastic Gaussian fields. Assuming exponential auto covariance kernels for aforementioned stochastic fields, they are discretized to two parts, including deterministic and stochastic parts employing Karhunen-Loeve theorem. Assuming linear form of strains, mechanical strains are defined applying first order shear deformation theory. Kinetic and potential energy of the composite plate is extracted using finite element formulation. Stochastic finite element formulation is derived employing Hamilton's principle and Euler-Lagrange and equations are verified with the results in the literature for deterministic case. After verification of formulation, material uncertainty effects on uncertainty of natural frequencies are investigated using Monte Carlo simulation. Results show that there is a linear relation between coefficient of variation of uncertain properties and coefficient of variation of stochastic natural frequencies. .

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1- Introduction

Composite structures have been widely replaced by heavy metals in aerospace and other industries because of their special properties. The characteristics such as high specific strength, the appropriate Young modulus are all due to the growing use of these materials in the construction of Aerospace structures and other industries. The features those are all associated with the low density of these materials. The type of process that takes place in the production of these materials (lamination, process of cooking, etc.) as well as the uncertainty arising from exposure of these materials to environmental conditions in the functional life cycle causes the statistical dispersion to increase in properties of these materials relative to other engineering materials. Because of dispersion in the mechanical properties of these materials such as Young's modulus, Poisson's ratio, shear modulus and rigidity, along with the exposing of these structures to thermal and mechanical loadings, the reliability estimation of these structures is a new challenge for the designers of composite structures. There are a lot of researches studying stochastic response of composite structures which can be categorized in two parts containing stochastic static response and stochastic dynamic response of composite structures. In the first category some researches study stochastic response of uncertain composite plate under deterministic loads [1-4] and the others study stochastic buckling of uncertain composite structures [5,6]. There are a lot of researches in the second category which study stochastic free and forced vibration of uncertain composite plates [7-11]. In aforementioned studies uncertainties in material properties are modeled as random parameters. In present study uncertainty propagation in free vibration of composite plate is studied assuming stochastic properties are Gaussian fields.

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2- Methodology

Material properties of composite plate including tensile modulus, shear modulus and density are assumed to be Gaussian stochastic field. Exponential auto-covariance kernels are considered for these stochastic fields as follows:

$$C_{E_{11}} = \sigma_{E_{11}}^2 \exp(-|(x-x')/l_x| - |(y-y')/l_y|) \tag{1}$$

$$C_{G_{12}} = \sigma_{G_{12}}^2 \exp(-|(x-x')/l_x| - |(y-y')/l_y|) \tag{2}$$

$$C_{G_{23}} = \sigma_{G_{23}}^2 \exp(-|(x-x')/l_x| - |(y-y')/l_y|) \tag{3}$$

$$C_{\rho} = \sigma_{\rho}^2 \exp(-|(x-x')/l_x| - |(y-y')/l_y|) \tag{4}$$

In which σ is standard deviation of stochastic variable, x, y are coordinates of the plate, l_x and l_y are correlation length of stochastic field in both direction. These stochastic fields can be decomposed to two parts including stochastic and deterministic parts applying Karhunen-Loeve theorem. For example for tensile modulus with auto-covariance kernel of Eq. (1) if stochastic field be defined in domain of Eq. (5) stochastic field can be decomposed in the form of Eq. (6) as follows:

$$x \in [-a,a], y \in [-b,b] \tag{5}$$

$$E_{11}(x,y,\omega) = \bar{E}_{11} + \sigma_{E_{11}} \sum_{i=1}^{n} \sqrt{\lambda_{x_i} \lambda_{y_i}}\, \zeta_i(\omega)\phi_i(x)\phi_i(y) \tag{6}$$

In which is average of tensile modulus over the lamina, and are Eigen values and Eigen vectors of Fredholm Eigen value problem respectively and are standard random variable with zero means and unit variances. This procedure can be found in Ghanem and Spanos book [12].

Deformation fields are defined by assuming first order shear deformation theory as Eq. (7):

$$\begin{aligned} u &= u_{\circ} + z\varphi_x \\ v &= v_{\circ} + z\varphi_y \\ w &= w_{\circ} \end{aligned} \tag{7}$$

Mechanical strains are derived employing definition of linear strains as Eqs. (5) to (12):

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{xx}^{(\circ)} \\ \varepsilon_{yy}^{(\circ)} \\ \varepsilon_{xy}^{(\circ)} \end{Bmatrix} + z \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \varepsilon_{xy}^{(1)} \end{Bmatrix} \tag{8}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \gamma_{yz}^{(\circ)} \\ \gamma_{xz}^{(\circ)} \end{Bmatrix} \tag{9}$$

$$\left\{ \varepsilon_{xx}^{(\circ)} \quad \varepsilon_{yy}^{(\circ)} \quad \varepsilon_{xy}^{(\circ)} \right\} = \left\{ u_{\circ,x} \quad v_{\circ,y} \quad u_{\circ,y} + v_{\circ,x} \right\} \tag{10}$$

$$\left\{ \varepsilon_{xx}^{(1)} \quad \varepsilon_{yy}^{(1)} \quad \varepsilon_{xy}^{(1)} \right\} = \left\{ \varphi_{x,x} \quad \varphi_{y,y} \quad \varphi_{x,y} + \varphi_{y,x} \right\} \tag{11}$$

$$\left\{ \gamma_{xz}^{(\circ)} \quad \gamma_{yz}^{(\circ)} \right\} = \left\{ \omega_{\circ,x} + \varphi_x \quad \omega_{\circ,y} + \varphi_y \right\} \tag{12}$$

Constitute equation of composite plate can be written as Eqs. (13) and (14):

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix}_m^k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix} \tag{13}$$

$$\begin{Bmatrix} \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix}^k = \begin{bmatrix} \bar{Q}_{44} & \bar{Q}_{45} \\ \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix}^k \begin{Bmatrix} \gamma^0_{yz} \\ \gamma^0_{xz} \end{Bmatrix} \tag{14}$$

Potential and Kinetic energy of the plate can be written as Eqs. (15) and (16) as follows:

$$U_p = \sum_{i=1}^{N\_layer} \frac{1}{2} \int_V \begin{pmatrix} \sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \sigma_{xy}\varepsilon_{xy} + \\ K_s\sigma_{yz}\varepsilon_{yz} + K_s\sigma_{xz}\varepsilon_{xz} \end{pmatrix}_i dV \tag{15}$$

$$T_P = \frac{1}{2}\int_V \rho\left[\dot{u}^2 + \dot{v}^2 + \dot{w}^2\right] dV \tag{16}$$

Applying finite element method stochastic equations of motion can be derived using Euler-Lagrange equation as Eq. (17):

$$\frac{\partial L}{\partial q_i}(t, q(t), \dot{q}(t)) - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i}(t, q(t), \dot{q}(t)) = 0 \quad i = 1,\dots,n \tag{17}$$

Stochastic discretized equation of motion is extracted in the form of Eq. (18):

$$\begin{aligned} &(\bar{M} + \sum_{i=1}^{n} M_i\zeta_i^{\rho})\{\ddot{X}\} + \\ &(\bar{K} + \sum_{i=1}^{n} K_i\zeta_i^{E_{11}} + \sum_{i=1}^{n} K_i\zeta_i^{G_{12}} + \sum_{i=1}^{n} K_i\zeta_i^{G_{23}})\{X\} = 0 \end{aligned} \tag{18}$$

Above equation is solved by generation standard random samples according to Monte Carlo simulation.

3- Results and Discussion

Stochastic free vibration of square composite plates with unit length, [0, 90, 90, 0] stacking sequences and different side to thickness ratios (a/h) are studied here. Average quantity of mechanical properties is presented in Table 1 as follows:

Tensile and shear modulus and density of the lamina are assumed to be stochastic Gaussian fields as Eq. (6).

Three terms in this equation is used to define stochastic fields and these fields are defined symmetrically. Correlation lengths of stochastic fields are equal to two in both directions. Converged results of Coefficient Of Variation (COV) of two first stochastic natural frequencies of plate with different a/h are plotted in Figs. 1 to 3 as follows:

As it can be seen from the above figures there is a linear relation between COV of natural frequencies and COV of random variables. There is a significant variation in natural frequencies due to uncertainty in mechanical properties. Therefore uncertainty in mechanical properties must be considered in order to predict natural frequencies and estimating structural reliability. Natural frequencies of thinner plates are more sensitive with uncertainty propagation in material properties. In order to study uncertainty propagation in each of random variables and its effects on COV of natural frequency, separately a stochastic analysis is done for plate with a/h=10 and results are presented Fig. 3:

Table 1. Properties of investigated lamina

properties	quantity
E_{11}/E_{22}	40
$G_{12}/E_{22} = G_{13}/E_{22}$	0.6
G_{23}/E_{22}	0.5
ν_{12}	0.25
$E_{۲۲}$ (GPa)	6.92

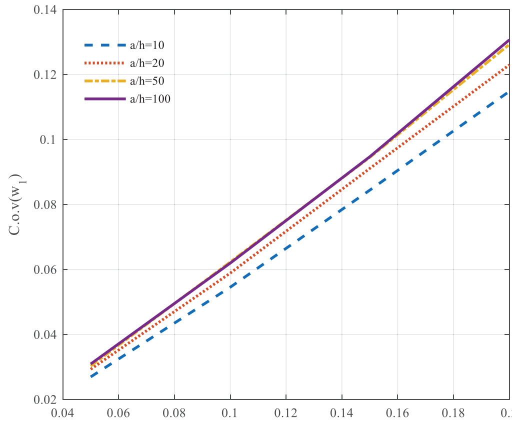

Fig. 1. COV of first natural frequency vs. COV of random variables

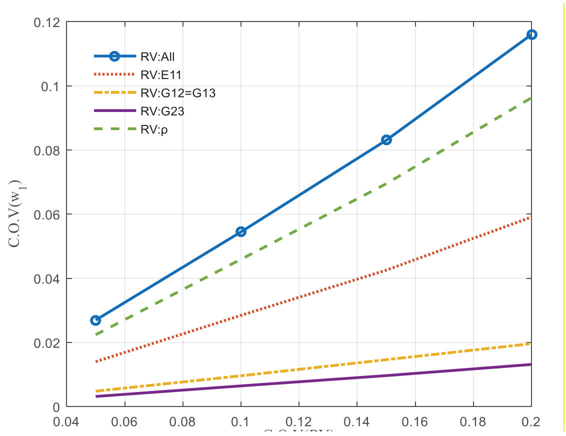

Fig. 3. COV of first natural frequency vs. COV of each random variables, separately

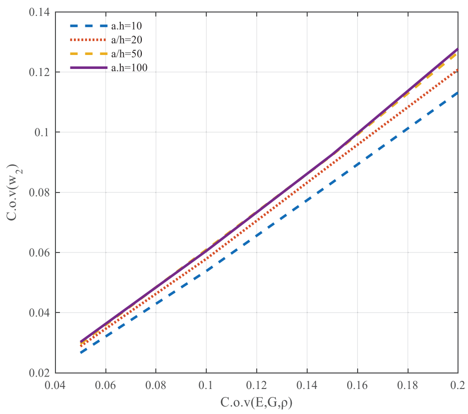

Fig. 2. COV of second natural frequency vs. COV of random variables

It can be seen, uncertainty of density and tensile modulus have the most effects on uncertainty of natural frequency. Also out of plane shear modulus has the minimum effects on uncertainty in natural frequency in comparison to other stochastic variables.

4- Conclusions

Stochastic free vibration of composite plate with spatially stochastic mechanical properties conducted using stochastic finite element method. Results show there is a linear relation between COV of natural frequencies and COV of random variables. Natural frequencies of thinner plates are more sensitive with uncertainty propagation in material properties. Above the stochastic mechanical properties, Density and tensile modulus have the most effects on dispersion of natural frequency.

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