Amirkabir Journal of Mechanical Engineering

Amirkabir J. Mech Eng., 53(special issue 2) (2021) 277-280
DOI: 10.22060/mej.2020.17112.6513

# The effect of accuracy of the length scale parameter on natural frequencies of porous rectangular microplate

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**ABSTRACT:** The length-scale parameter as a primary factor has an important role in the approximation of natural frequencies of micro-structures. Applying the exact length scale parameters which are recently determined by researchers, natural frequencies of porous microstructures are determined. Since the assumption of constant length-scale parameter leads to deviation of natural frequencies from their exact value, this research applies a length scale parameter which is a function of plate thickness and material. To model the porous structure of the microplate, various porous models including evenly porosity mode, unevenly symmetric porosity model, and unevenly asymmetric porosity model are employed. The microplate is assumed to be thin, and classical plate theory is utilized to approximate the displacement field of the microplate. The modified couple stress theory is used to capture the microstructural behavior of the microplate. The trial functions which satisfy the boundary conditions are taken as the polynomial form. Evaluating the energy values of the system, the Rayleigh-Ritz method is employed to solve the governing equations of the system. The results obtained in the present work are validated with data given in the literature search. A parameter study is performed to study the effects of various parameters on the natural frequency of the microplate.

**Review History:**

Received: Sep. 26, 2019
Revised: Dec. 01, 2019
Accepted: Dec. 29, 2019
Available Online: Feb. 17, 2020

**Keywords:**

Porous Rectangular Microplate
Modified Couple Stress Theory
Length Scale Parameter

## 1. INTRODUCTION

Non-local theory [1-3], strain gradient theory [4], coupled stress theory [5, 6], and modified coupled stress theory [7] are common and useful non-classical theories that are used to capture the size effects in microstructures. In most studies that are performed on microplates, a constant length scale parameter is assumed. Akbarzadeh khorshidi [8] experimentally extracted the exact values of the length scale parameter for several metals and showed that the length scale parameter is a function of the thickness as well as the material. Porous materials have a two-phase structure with one phase being solid (body) and the other being liquid or gas. The industry-wide applications of porous materials include filters, air conditioners, separators, heat exchangers and refrigerants. The present study applies the length scale parameter which is function of thickness. Natural frequencies of the microplate are simulated based on a new non-uniform model. The Lagragnian function is minimized applying the Ritz method to study the free vibration of the microplate.

## 2. MATHEMATICAL FORMULATION

The modulus of elasticity and density for the porous rectangular microplate are determined by the following models:

$$E(z) = E_0\left[1 - e_0\varphi(z)\right] \tag{1}$$

$$\rho(z) = \rho_0\left[1 - e^*\varphi(z)\right] \tag{2}$$

In these equations, $\varphi(z)$ is the porosity distribution function, $e_0$ is the porosity of the plate, and $e^*$ is the density parameter. The parameter $e^*$ is defined as $e^* = 1 - \rho_0/\rho_1$, and its value is between zero and one. Parameters $E_0$ and $E_1$ are the maximum and minimum values of elastic modulus of the plate and are related to $\rho_0$ and $\rho_1$. The porosity distribution function is expressed by following four different models:

$$\varphi(z) = \begin{cases} 1 & \text{Type I} \\ \dfrac{1}{e_0} - \dfrac{1}{e_0}\left(\dfrac{2}{\pi}\sqrt{1-e_0} - \dfrac{2}{\pi} + 1\right)^2 & \text{Type II} \\ Cos\left(\pi z / h\right) & \text{Type III} \\ Cos\left(\pi z / h + \pi/4\right) & \text{Type IV} \end{cases} \tag{3}$$

According to this non-classical theory, the coupling stress tensor is symmetric and contains one length scale parameter. The strain energy for a linear elastic microplate is determined by the following equation;

$$U_s = \frac{1}{2}\int_0^a\int_0^b\int_{-h/2}^{h/2}\begin{pmatrix} \sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \tau_{xy}\gamma_{xy} + \\ m_{xx}\chi_{xx} + m_{yy}\chi_{yy} + 2m_{xy}\chi_{xy} + \\ 2m_{xz}\chi_{xz} + 2m_{yz}\chi_{yz} \end{pmatrix} dzdydx \tag{4}$$

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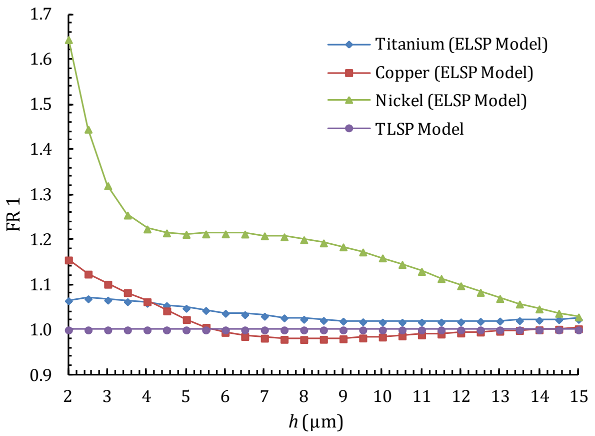


**Fig. 1.Variation of *FR1* versus the thickness of the porous microplate**

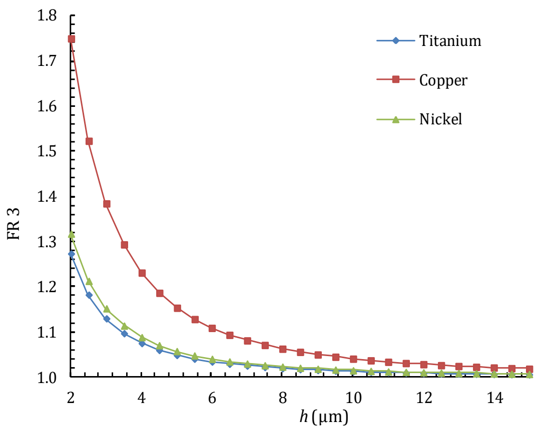


**Fig. 2.Variation of *FR3* versus the thickness of the porous microplate**

where $\sigma$ is the stress tensor; $\varepsilon$ is the strain tensor; $m$ is the deviatoric part of the symmetric couple stress tensor, and $\chi$ is the symmetric curvature tensor. The kinetic energy of micro-plate for classical plate theory is expressed as follows for stress couple theory:

$$T_P = \frac{1}{2}\int_0^a\int_0^b\int_{-h/2}^{h/2} \rho(z)\left[\dot{U}^2 + \dot{V}^2 + \dot{W}^2\right]dzdydx \tag{5}$$

The Lagrangian function for the system is

$$\Pi = \sum U_{\max} - \sum T_{\max} \tag{6}$$

The following derivatives are employed to minimize Eq. (6):

$$\frac{\partial \Pi}{\partial a_n} = \frac{\partial \Pi}{\partial b_n} = \frac{\partial \Pi}{\partial c_n} = 0 \tag{7}$$

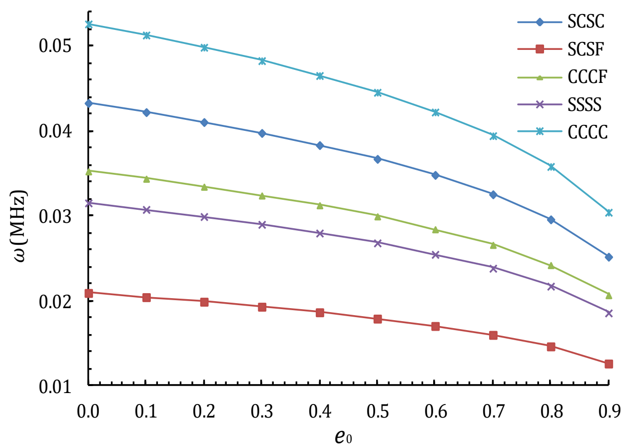


**Fig. 3. Variation of natural frequencies (MHz) of epoxy porous microplate versus the porosity parameter for a porous model of type I and various boundary conditions**

## 3. RESULTS AND DISCUSSION

In this section, the natural frequency values of rectangular microplates for various porous models are obtained. To this aim, the following frequency ratios are employed:

$$FR1 = \frac{\omega_{ELSP}}{\omega_{TLSP}}, FR3 = \frac{\omega_{TLSP}}{\omega_{CPT}} \tag{8}$$

where $\omega_{ELSP}$ and $\omega_{TLSP}$ represent the natural frequencies of microplate which are obtained by exact length scale parameter and traditional length scale parameter, respectively. Also, $\omega_{CPT}$ is the natural frequency of the system is determined by neglecting the microstructural effects. Fig. 1 shows the variation of *FR1* with respect to the thickness of a porous microplate made of titanium, copper, and nickel. The assumptions in this form are $a/h = 20$, $a/b = 20$, and $e_0 = 0.9$. All edges of the plate are simply supported. As the Figure shows, natural frequency obtained by traditional length scale parameter is constant, while application of exact length scale parameter results in deviation from the traditional model. The Figure shows that the deviation of the frequency values for nickel is very high. Also, by increasing the thickness values, these deviations are decreased.

Fig. 2 shows the variation of *FR3* versus the thickness of the porous microplate for various types of metals. As it can be seen, *FR3* is always higher than one. So, considering the length scale parameter increases the natural frequencies of the microplate. Also, the curves converge to one by increasing the thickness, which means that for $h > 20\mu m$ microstructural are lost; thus application of non-classical theory is not required for $h > 20\mu m$. Fig. 3 shows the variation of natural frequencies (MHz) of epoxy porous microplate; the porous model with type I is assumed to obtain the results for various boundary conditions; the Figure shows that increasing the porosity parameter decreases the natural frequency of the microplates. Absolute values of the curve slopes gradually decrease and the frequency reduction rate becomes more pronounced.

## 4. CONCLUSIONS

Using the traditional length scale parameter with value results in incorrect natural frequencies. Using the traditional model, the deviation of natural frequencies from the exact model is significant and relatively high for nickel microplate. This is due to incorrect estimation of the length scale parameter for this metal. Application of non-classical theories is necessary to calculate frequency values in plates with micro- and nano-thicknesses, but the thickness range for mandatory application of non-classical theories is different for various materials. The porosity effect for uniform models reduces the natural frequency of micro plates; while for non-uniform models, the behavior of natural frequencies of the microplates is not generally predictable. The natural frequency values of the microplate with four-sided clamp boundary conditions are higher than other boundary conditions; this is due to the increased rigidity of the microplate.

## REFERENCES

[1] A.C. Eringen, Nonlocal polar elastic continua, International journal of engineering science, 10(1) (1972) 1-16

[2] K. Khorshidi, A. Fallah, A. Siahpush, Free vibrations analaysis of functionally graded composite rectangular nanoplate based on nonlocal exponential shear deformation theory in thermal environment, (2017).

[3] K. Khorshidi, A. Siahpush, A. Fallah, Electro-Mechanical free vibrations analysis of composite rectangular piezoelectric nanoplate using modified shear deformation theories, Journal of Science and Technology of Composites, 4(2) (2017) 151-160

[4] D.C. Lam, F. Yang, A. Chong, J. Wang, P. Tong, Experiments and theory in strain gradient elasticity, Journal of the Mechanics and Physics of Solids, 51(8) (2003) 1477-1508.

[5] R.A. Toupin, Elastic materials with couple-stresses, Archive for Rational Mechanics and Analysis, 11(1) (1962) 385-414

[6] R. Mindlin, H. Tiersten, Effects of couple-stresses in linear elasticity, Archive for Rational Mechanics and analysis, 11(1) (1962) 415-448.

[7] F. Yang, A. Chong, D.C.C. Lam, P. Tong, Couple stress based strain gradient theory for elasticity, International Journal of Solids and Structures, 39(10) (2002) 2731-2743.x

[8] M.A. Khorshidi, The material length scale parameter used in couple stress theories is not a material constant, International Journal of Engineering Science, 133 (2018) 15-25.

[9] M. Şimşek, M. Aydın, Size-dependent forced vibration of an imperfect functionally graded (FG) microplate with porosities subjected to a moving load using the modified couple stress theory, Composite Structures, 160 (2017) 408-421.

[10] J. Zhao, Q. Wang, X. Deng, K. Choe, R. Zhong, C. Shuai, Free vibrations of functionally graded porous rectangular plate with uniform elastic boundary conditions, Composites Part B: Engineering, 168 (2019) 106-120.

**HOW TO CITE THIS ARTICLE**

*A. Bakhsheshy, H. Mahbadi, The effect of accuracy of the length scale parameter on natural frequencies of porous rectangular microplate. Amirkabir J. Mech Eng., 53(special issue 2) (2021) 277-280.*

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