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# Free vibration analysis of functionally graded carbon nanotubes reinforced composite skew folded plates using the isogeometric approach

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ABSTRACT: In this research, an approach based on the isogeometric method is developed to study the free vibration behavior of functionally graded carbon nanotubes reinforced composite skew folded plates. In this method, non-uniform rational B-splines basis functions are used for approximation of the geometry as well as the displacement field. The plates are reinforced by single-walled carbon nanotubes which are assumed to be graded through the thickness direction with different distribution patterns. The effective mechanical properties of composite skew folded plates are captured by the modified rule of mixtures approach. Modeling of the skew folded plate is accomplished by two non-uniform rational B-splines patches which is one of the strengths of the research. The equations of motion of each patch are derived based on classical plate theory and then are discretized using non-uniform rational B-splines basis functions. The final form of the discretized equations is generated after the transformation of the element matrices of each patch and then applying the continuity conditions along the boundary of the patches with the aid of the bending strip method. Afterward, several numerical examples are provided to prove the accuracy and reliability of the proposed formulation. The results exhibit that the present approach can precisely predict the natural frequencies of skew folded plates with a low computational cost. Eventually, a set of new results are presented for different geometrical and material parameters of the skew folded plate.

# **1- Introduction**

Nowadays, skew plates are widely used as the structural element of many modern structures. Moreover, folded plates are an appropriate candidate for use in different engineering applications. Meanwhile, several studies have been performed on the improvement of reinforced composite materials during recent years. Due to the excellent mechanical, thermal and electrical properties of Carbon NanoTubes (CNTs), they are a suitable choice as a reinforcement phase in a polymer matrix. The distribution of CNTs in a matrix may be uniform or Functionally Graded (FG) in the thickness direction of the plate. The latter case is referred to as Functionally Graded Carbon NanoTube Reinforced Composite (FG-CNTRC) skew folded plates.

Numerous studies have been executed to investigate the free vibration behavior of rectangular folded plates. Most of these studies have been accomplished in the last two decades, along with the development of high-speed computers. In this regard, various numerical techniques have been proposed to study the mechanical behaviors of folded plates. These methods are finite strip method [1], combined boundary element-transfer matrix method [2], mesh-free Galerkin method [3], Finite Element Method (FEM) [4], and isogeometric method [5].

The IsoGeometric Analysis (IGA) is a powerful numerical technique that was firstly proposed by Hughes et

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al. [6]. In this method, the exact geometric description is used to approximate the solution field. Since the introduction of the IGA, it has been extensively employed to analyze FG-CNTRC structures.

#### 2- Solution Method

In this study, it is assumed that each patch of the skew folded plate is fabricated from a polymer matrix reinforced by Single-Walled Carbon NanoTubes (SWCNTs) with Uniform Distribution (UD) as well as three FG distributions defined as FG-X, FG-O, and FG-V. The effective mechanical properties of FG-CNTRC skew folded plates are estimated via a modified rule of mixture.

To derive the equations of motion of each patch, the local coordinate system is placed in the midplane corner of the patch. The displacement field of each patch is approximated based on the Classical Plate Theory (CPT). The weak form for free vibration analysis is obtained using the principle of virtual work. Then the field equations are discretized using Non-Uniform Rational B-Splines (NURBS) basis functions. Afterward, the element matrices which are evaluated in the local coordinate system of the patch, are transferred to the global coordinate system. Moreover, the bending strip method [7] is used to define the continuity conditions along the intersection of the patches.

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#### **3- Results and Discussion**

As a part of the validation study, the non-dimensional fundamental frequency parameter  $(\overline{\omega}_1)$  of simply supported FG-CNTRC square plates with L/h = 50 are listed in Table 1. The results are prepared for various distributions of CNTs as well as different volume fractions. The presented data are compared with FSDT based solution conducted by Zhu et al. [8]. They employed FEM to extract the results. According to provided data in Table 1, it can be observed that the convergence behavior is very good. Moreover, both sets of results exhibit a very good agreement.

Fig. 1 illustrates the variation of the fundamental frequency ratio ( $\beta_1 = \omega_1^{\text{skew}} / \omega_1^{\text{rectangular}}$ ) with respect to the skew angle ( $\theta$ ) for simply supported and fully clamped FG-X CNTRC skew folded plates. it can be easily deduced that the fundamental frequency ratio is always greater than unity, which means that skew folded plates have greater fundamental frequency compared to the rectangular ones.

### **4-** Conclusions

In this paper, the IGA is employed to study free vibration behavior of FG-CNTRC skew folded plates. The equivalent mechanical properties of the plate are approximated according to the modified rule of mixture. The skew folded plate is modeled by two patches. The governing equations of each patch are derived with the aid of the principle of virtual work based on the CPT. After an appropriate coordinate transformation, the bending strip method is applied to fulfill the continuity conditions. Several numerical examples are presented to show the efficacy of the proposed formulation and to discuss the effect of related parameters. It is observed that, for all distribution patterns, with increasing CNTs volume fraction, the fundamental frequency of skew folded plate increases. Moreover, among all considered distribution patterns, FG-X and FG-O shapes give the highest and lowest frequencies. Furthermore, it is concluded that the skew angle has a pronounced effect on the computed results.

Table 1. Convergence and comparison study of non-dimensional fundamental frequency parameter  $(\bar{\omega}_1 = \omega_1 (L^2/h) \sqrt{\rho^m / E^m})$  for various types of simply supported FG-CNTRC square plates with different CNTs volume fractions,  $(\alpha = 180^\circ, \theta = 0^\circ, L/h = 50)$ .

| $V_{CNT}^*$ | Method         | $N_{\xi}=N_{\eta}$ | UD          | FG-V    | FG-O    | FG-X    |
|-------------|----------------|--------------------|-------------|---------|---------|---------|
| 0.11        | Present (CPT)  | 1                  | 19.582<br>6 | 16.4483 | 14.4294 | 23.6461 |
|             |                | 3                  | 19.581<br>3 | 16.4471 | 14.4284 | 23.6446 |
|             |                | 5                  | 19.581<br>3 | 16.4471 | 14.4284 | 23.6446 |
|             |                | 7                  | 19.581<br>3 | 16.4471 | 14.4284 | 23.6446 |
|             |                | 9                  | 19.581<br>3 | 16.4471 | 14.4284 | 23.6446 |
|             | FEM (FSDT) [8] |                    | 19.223      | 16.252  | 14.302  | 22.984  |
| 0.14        |                | 1                  | 21.897<br>4 | 18.3017 | 16.0070 | 26.5209 |
|             |                | 3                  | 21.895<br>9 | 18.3004 | 16.0059 | 26.5192 |
|             |                | 5                  | 21.895<br>9 | 18.3004 | 16.0059 | 26.5192 |
|             |                | 7                  | 21.895<br>9 | 18.3004 | 16.0059 | 26.5192 |
|             |                | 9                  | 21.895<br>9 | 18.3004 | 16.0059 | 26.5192 |
|             | FEM (FSDT) [8] |                    | 21.354      | 17.995  | 15.801  | 25.555  |
| 0.17        |                | 1                  | 24.117<br>3 | 20.2095 | 17.6958 | 29.1809 |
|             |                | 3                  | 24.115<br>7 | 20.2080 | 17.6945 | 29.1791 |
|             |                | 5                  | 24.115<br>7 | 20.2080 | 17.6945 | 29.1791 |
|             |                | 7                  | 24.115<br>7 | 20.2080 | 17.6945 | 29.1791 |
|             |                | 9                  | 24.115<br>7 | 20.2080 | 17.6945 | 29.1791 |
|             | FEM (FSDT) [8] |                    | 23.697      | 19.982  | 17.544  | 28.413  |



Fig. 1. Variation of fundamental frequency ratio ( $\beta_1$ ) with respect to skew angle ( $\theta$ ) for simply supported and fully clamped FG-X CNTRC skew folded plates with three different crank angles, ( $V_{CNT}^* = 0.17$ , L/h = 50).

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