Reconstruction of Electrical Tomography Images based on Parameter Estimation Method in Inverse Heat Transfer

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ABSTRACT

Electrical tomography is a non-invasive method that is used to visualize the internal structure of an object by applying voltage or current and using an image reconstruction algorithm. In spite of some advantages such as simplicity and low cost, the images of tomography systems have low resolution and quality. In addition to system hardware that causes errors, the reason behind the low-quality images is the reconstruction algorithms. In this research, the idea of image reconstruction by solving the heat conduction equations instead of solving electrical equations is used and the thermal conductivity distribution is calculated. For this purpose, the temperature of the active surface is changed and the generated heat flux between the active and other surfaces is measured. The Levenberg-Marquardt algorithm is employed to estimate the geometric parameters of the unknown objects. Three different test cases are selected to investigate the capability of the proposed algorithm in the estimation of the unknown geometries. The results show that the proposed algorithm has the ability to estimate unknown shapes. Sensitivity analysis is also performed in order to examine the effect of noise in the detection of the unknown geometry. The results show that with increasing the value of noise, the shape estimation error increases, but the shape has a good agreement with the original geometry. Also, results of choosing different combinations of active surfaces to create thermal flux show that in addition to the effectiveness of active surfaces in increasing the accuracy of shape estimation, the unknown geometry can be estimated using two thermal flux measurements.

KEYWORDS

Electrical tomography, Image reconstruction, Inverse heat transfer, Levenberg – Marquardt algorithm, Parameter estimation.

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1. Introduction

After the discovery of x-rays, physicians and treatment centers turned to the use of this method for imaging from inside the body and diagnosis of diseases. This method is called tomography which is used to determine the internal structure of objects. The electrical tomography method is popular due to being non-invasive and contactless. Electrical tomography is one of the methods used to identify the internal structure of objects. Contrary to methods which employ the transmitting signals or radiating rays for imaging, this technique examines the internal tissue of the body by using the difference of electrical properties in the material. Compared to the X-ray method, the electrical technique has several advantages such as low cost, eliminating harmful radiation and etc. Besides these advantages, the method also has some drawbacks such as low quality and low resolution.

The main components of an electrical tomography system include mounted sensors on the external surface of test object, a processor and a data acquisition system. The schematic of an electrical tomography system is shown in Figure 1.

![Figure 1. Electrical tomography system](image)

In this system, by applying the electrical voltage or current to the mounted electrodes on the external wall of the object, the distribution of permittivity inside the object is calculated to determine its structure.

One of the factors that affect the quality of produced images in the tomography method is the reconstruction algorithms. Previous investigations introduced and categorized the image reconstruction algorithms into two groups of iterative methods (Landweber method, Newton-Raphson method) and non-iterative methods (linear back projection, singular value decomposition, Tikhonov method [1-3]). The reconstructed images by previous algorithms have low quality and accuracy.

The inverse heat transfer problems belong to the category of inverse problems. The inverse heat transfer algorithms not only determine the unknown boundary conditions but also estimate the physical properties [4, 5] and the geometrical shape of objects [6, 7]. Therefore, the inverse heat transfer problem is applicable to the electrical tomography system in order to estimate the distribution of thermal conductivity and to identify the internal structure of the body. The governing equations of electrical capacitance tomography are similar to heat conduction. Therefore, instead of solving the electrical equations, the governing equations of heat conduction are used.

In this paper, an image reconstruction algorithm based on inverse heat transfer problems is presented to identify the internal structure of the objects. For this purpose, the image reconstruction process using the Levenberg - Marquardt algorithm and the method of changing the physical properties are implemented in the unstructured mesh.

2. Methodology

The Levenberg-Marquardt algorithm is a parameter estimation method and based on the reduction of the difference between the measured and estimated thermal heat flux according to Eq. (1).

\[
S = \sum_{i=1}^{M} \left[ Y_i - Q_i(P) \right]^2
\]  

(1)

where \( Y \) and \( Q \) are the exact and estimated heat flux vectors. The Levenberg-Marquardt equation for estimating the unknown parameters vector \( (P) \) is presented in Eq. (2).

\[
P^{h+1} = P^h + \left( X^T X + \mu \Omega \right)^{-1} X^T \left( Y - Q \left( P^h \right) \right)
\]  

(2)

where \( X \) is the sensitivity matrix. In the iterative algorithm, the unknown parameters which include the coordinates of the area center \( (x_c, y_c) \) and the distance of the control points from the center of the object \( (R) \) are estimated in each iteration in order to determine the shape.

3. Image Reconstruction Algorithm

Steps for reconstructing the images are as follows:

1. Choosing a circle in the center of the domain as an initial guess.
2. Estimation of new parameters \( P_{new} \) using Eq. (2).
3. Drawing the initial shape using a spline passing through the new control points [8].
4. Smooth the initial geometry in step 3 using a smoothing algorithm to prevent the formation of a curve with sharp edges [9].
5. Changing the physical properties of computational cells inside the curve
6. Changing the temperature of the active surfaces and calculating the thermal heat flux passing through the surfaces and the determine \( S(P^h) \).

The steps of the introduced algorithm continue until the convergence is achieved.
The winding number $\omega$ is used to determine the points inside the curve and change the thermal conductivity according to Eq. (3).

\[
\omega(x_0,y_0) = \frac{1}{2\pi} \int \frac{(\psi_x(t) - x_0)\psi_y'(t) - (\psi_y(t) - y_0)\psi_x'(t)}{\sqrt{(\psi_x(t) - x_0)^2 + (\psi_y(t) - y_0)^2}} \, dt
\]

Figure 2 shows the schematic of changing the thermal conductivity and estimating the unknown geometry.

\[
\omega(x_0,y_0) = \frac{1}{2\pi} \int \frac{(\psi_x(t) - x_0)\psi_y'(t) - (\psi_y(t) - y_0)\psi_x'(t)}{\sqrt{(\psi_x(t) - x_0)^2 + (\psi_y(t) - y_0)^2}} \, dt
\]

4. Results and Discussion

In order to evaluate the accuracy of the algorithm, several test cases are considered. One of the test cases is shown in Figure 3. To create thermal heat flux inside the object, first the temperature of the surfaces 4 & 5 and then the surfaces 6 & 7 is changed. The image reconstruction results show that the geometrical shape is estimated after 10 iterations and with the maximum error of 4%.

Due to the sensitivity of inverse problems to input parameters, the second geometry is considered to examine the effect of the input errors in the shape estimation. The geometry shown in Figure 4 is considered. Two surfaces 5 and 7 are activated sequentially with different temperature. The measurements are changed by adding random errors with an amplitude of 10% and 30%. The results of the geometric shape estimation show that increasing the measurement error, in addition to increments the image reconstruction error, leads to an increase in the number of iterations.

5. Conclusions

In this paper, an image reconstruction algorithm for the electrical tomography system based on the Levenberg-Marquardt algorithm is presented. For this purpose, the method of changing physical properties is used to identify the geometric shape of the unknown object. Therefore, after estimating the geometric shape in each iteration, the winding number is used to change the thermal conductivity of the computational cells inside the closed curve. The results show that the proposed algorithm is applicable in the tomography system in order to identify the internal structure of the object. Furthermore, the results of the sensitivity analysis show that despite increasing the image reconstruction error due to increasing the noise in the measurements; the unknown shape can be estimated with little deviation from the exact one.

6. References


