Three-dimensional Elastic-Plastic Deformation Analysis of Composite Sandwich Panel under blast loading

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ABSTRACT

A numerical analysis base on the three-dimensional elasticity solution is presented for predicting the plastic deformation of a cylindrical composite sandwich panel. An extended non-linear higher-order sandwich panel theory is applied to model core compressible effect. The non-linear governing partial differential equations of motion are discretized and reduced to ordinary differential equations by applying differential quadrature method (DQM) and solved using Newmark method. The effects of various parameters like that panel dimensions, layers thickness, pulse duration and maximum pressure on the plastic deformation of the panel was investigated. The obtained results using the present method are compared with finite element solutions by commercial software ANSYS and good agreement is demonstrated. It is observed that significant less computational time and hardware capacity for the proposed method with respect to the finite element solution is require. It was shown that inner layer of sandwich panels are supposed to higher stresses and are more likely places for panel failure.

KEYWORDS

Sandwich panel, plastic deformation, facesheet fracture, blast loading, DQ-Newmark.

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1. Introduction

Composite and sandwich panels consist of fiber-reinforced polymers facesheets and polymeric foam cores may be exposed to a pulse pressure loading created by the explosion or blast and response of the panel can involves plastic deformation and facesheets fracture. Numerical and experimental studies done lately have shown that elastic-plastic deformation is the main phenomenon that takes place in the foam cores used nowadays in sandwich structures [1-2]. Wang et al. [3] have done experimental test on the step-wise graded cores to fine that the effects of staggering the foams on the blast resistance of composite sandwich panels. They reported that the blast strength of a composite sandwich panel can be increase by interchanging the foams such that the softest core, which would experience significant core crushing and plasticity, is the most proximate layer to shock wave from a blast. Elastic-plastic deformation of a double curved composite sandwich panel under blast load was studied by Hoo Fatt and Surabhi [4].

2. Governing Equations of Motion

Consider a simply supported cylindrical sandwich panel with the radii of mid-surface $a_c$, $a_t$, and $a_b$, panel angle $\beta$, total thickness $H= h_t + h_b + h_c$, length $L$ and span $b$ as shown in Fig. 1. It should be mentioned that the sub-indices $t$, $c$, and $b$ indicate the top, core, and bottom layers, respectively. The panel is subjected to the uniformly distributed external pressure pulse loading with the below equation:

$$P(t) = P_0\left(1 - \frac{t}{t_d}\right)e^{\alpha t} \quad (1)$$

in which $P_0$, $t_d$, and $t$ are maximum pressure, pulse decay and time, respectively. The core of the sandwich shell is made of isotropic viscoelastic material, such as PVC foam. Likewise, it is assumed that the top and bottom layers are made of Woven Roving E-Glass/Vinyl Ester with 0-degree orientation with respect to the hoop direction.

The facesheets are assumed to satisfy the Kirchhoff-love assumptions and a non-linear fourth-order function is used to approximate displacement fields of the core layer as considered in [5]. The three-dimensional governing dynamic equilibrium equations of panel are as follows:

$$\begin{align*}
\frac{\partial \sigma_{xx}}{\partial z} + \frac{1}{z + a_t} \frac{\partial }{\partial \theta} \left( \frac{a_t}{z + a_t} \sigma_{\theta\theta} \right) - \sigma_{zz} &= \rho \frac{\partial^2 u(x, \theta, z)}{\partial t^2} \\
\frac{\partial \sigma_{zz}}{\partial z} + \frac{1}{z + a_t} \frac{\partial }{\partial \theta} \left( \frac{a_t}{z + a_t} \sigma_{\theta\theta} \right) + \frac{2\sigma_{zz}}{z + a_t} &= \rho \frac{\partial^2 v(x, \theta, z)}{\partial t^2} \\
\frac{\partial \sigma_{xx}}{\partial z} + \frac{1}{z + a_c} \frac{\partial }{\partial \theta} \left( \frac{a_c}{z + a_c} \sigma_{\theta\theta} \right) + \frac{2\sigma_{xx}}{z + a_c} &= \rho \frac{\partial^2 w(x, \theta, z)}{\partial t^2}
\end{align*} \quad (2)$$

Integrating Eq. (2) in radial direction, the seven partial governing differential equations of motion can be written in terms of the displacement fields ($u'$, $v'$, $w'$, $u''$, $v''$, $w''$). In this paper assumed that panel facesheets undergoing pure elastic deformations and core has elastic-perfectly plastic properties.

![Figure 1. Geometrical properties and Schematic view of the cylindrical sandwich panel](image)

In order to describe plastic yielding in the core, the criterion introduced in [6] is applied:

$$\sigma_{\text{von}} = \sigma_y \quad (3)$$

where $\sigma_{\text{flow}}$ and $\sigma_y$ is the flow and effective stress, respectively. Also, the modified Hashin-Rotem failure criterion is applied for orthotropic facesheets failures examine:

$$\begin{align*}
\sigma_{\text{ij}} &= 1 \quad \text{if} \quad \sigma_{\text{ij}} > 0 \\
\sigma_{\text{ij}} &= 1 \quad \text{if} \quad \sigma_{\text{ij}} < 0 \\
\sigma_{\text{ij}} &= 1 \quad \text{if} \quad \sigma_{\text{ij}} > 0 \\
Y_c &= 1 \quad \text{if} \quad \sigma_{\text{ij}} < 0
\end{align*} \quad (4)$$

In this paper, the combined DQ-Newmark method is used to discrete and solve three-dimensional partial differential equation of motion of the sandwich panel. The polynomial based differential quadrature as a semi-analytical method is introduced to approximate the first and the second order derivatives of the function and discretize the governing equations:

$$\left[ \frac{\partial f(x, \theta)}{\partial x} \right]_{x=t_i} = \sum_{j=1}^{N} a_i^j \cdot f(x_i, \theta_j)$$

for $i = 1, 2, \ldots, N$

and

$$\left[ \frac{\partial f(x, \theta)}{\partial \theta} \right]_{\theta=\theta_i} = \sum_{j=1}^{Q} b_i^j \cdot f(x, \theta_j)$$

for $j = 1, 2, \ldots, M$

where $a_i^{(l)}$ and $b_i^{(l)}$ are the weighting coefficients for the first derivative and $N$ and $Q$ denote the number of
sampling points in r and θ directions, respectively. Dynamics partial differential equations converted to linear system with 7×N×M ordinary differential equations using DQ method and are written after discretization as follows:

\[
[M]_{7×N×M} \begin{bmatrix} \dot{X}_1 \\ \vdots \\ \dot{X}_N \end{bmatrix}_{7×N×M} + \begin{bmatrix} K \end{bmatrix}_{7×N×M} \begin{bmatrix} X_1 \\ \vdots \\ X_N \end{bmatrix}_{7×N×M} = \{p(t)\} \tag{6}
\]

where \([M]\) and \([K]\) are the mass and stiffness matrices, respectively and \(\{X\}\) denote displacement fields at three directions of panel.

In next, an implicit approach namely "Newmark" method is used to solve the set of ordinary differential equation of sandwich panel.

3. Results and Discussion

Table 1 depicts material properties of Woven Roving E-Glass/Vinyl Ester and PVC foam that are considered for facesheets and core.

<table>
<thead>
<tr>
<th>Table 1. Material properties of the sandwich panel</th>
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<tbody>
<tr>
<td>E-Glass/ Vinyl Ester</td>
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<tr>
<td>E1(Gpa)</td>
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<tr>
<td>E2(Gpa)</td>
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<tr>
<td>E3(Gpa)</td>
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<tr>
<td>G12(Gpa)</td>
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<td>G21(Gpa)</td>
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<td>G31(Gpa)</td>
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<td>ν12</td>
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<td>ν23</td>
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<tr>
<td>ν31</td>
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<tr>
<td>ρ(Kg/m³)</td>
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<tr>
<td>σY(MPa)</td>
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<td>τY(MPa)</td>
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A composite sandwich panel with \(L=b(β×a_c)=0.5 \text{ m}, R_1=1 \text{ m}, h_t=h_b=0.005 \text{ m}, h_c=0.04 \text{ m}\) made of E-glass/Vinyl Ester facesheets and a PVC H250 foam core under pulse pressure with \(P_0=2 \text{ MPa}\) and \(t_0=0.002 \text{ s}\) is chosen to compare the elastic-perfectly plastic response with the finite element solution using ANSYS commercial software. Von-Mises stress of core for various times is plotted in Figs. 11. Accordingly, good agreement between results obtained by DQM-Newmark method and those predicted by finite element solution is observed.

Figs. 3 and 4 shows the peak pressure corresponded to initial core yield versus panel mid-surface radius and pulse decay for the various core materials. The peak pressure to plasticity decreases with increasing mid-surface radius and pulse decay.

4. Conclusions

In this paper, plastic deformation and failure analysis of the cylindrical sandwich panel under the lateral external blast has been carried out. It is assumed that the panel has simply supported boundaries at the four edges and an elastic-perfectly plastic model is used for modeling the mechanical behavior of the core material. A nonlinear higher order core theory is applied to consider core compressibility effects and three-dimensional governing equations of motion are discretized and solved using a new method combination of DQ and Newmark methods. It is observed that the programming code is faster and requires 50 percent less time than finite element solution in ANSYS.
5. References


