Optimal Control Based on Minimum-Energy Trajectory Planning of a Quadrotor

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ABSTRACT
Quadrotors have high energy consumption, hence minimizing their energy consumption play a crucial role in terms of enhancing their operational range and flight time. In this paper, optimal control based on a minimum-energy trajectory planning algorithm has introduced between two positions to maximize operation time. To do this, first, dynamic equations of quadrotor and Brushless motor are derived. Energy consumption of quadrotor is introduced as a cost function and the minimum energy path is determined using optimal control theory. All constraints are combined with the Hamiltonian equation using Lagrange multiplier. Finally, simulation results are compared with results of conventional trapezoidal velocity profile which shows energy saving up to 4%. Also, results reveal that the influence of operation time is far more than path length on energy consumption. In order to verify the validity of the simulation results, they are compared with the results of an experimental model which is consisting brushless motor, sensor and control board. As well as using simulation results in different situations, a mathematical equation was extracted among path length, operation time and energy consumption which can be useful to estimate the maximum flight range or operation time considering the amount of energy of the battery.

KEYWORDS
Quadrotor, minimum energy trajectory, optimal control, Hamiltonian equations

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1. Introduction

High energy consumption is one of the most important challenges about quadrotors [1]. Trapezoidal velocity profile is a common method which is used widely in all types of robots. The major problem of such approach is severe dependency to acceleration/deceleration rates that leads to decrease in energy efficiency of quadrotor [2]. For this reason, minimum energy trajectory is the best strategy to enhance the energy efficiency [3].

The paper aimed to minimum energy path planning and propose an optimal controller so that quadrotor can reach the goal as well as satisfying its constraints. Considering the detailed model of motor and its ingredients, verifying validity of simulation results by experimental data and also deriving the mathematical relation among duration flight, maximum flight range and energy consumption are contributions of the paper.

2. Metodology

Dynamic model of the quadrotor is as following [4]:

\[
\begin{align*}
\dot{X} & = \left( \sin \psi \sin \varphi + \cos \psi \sin \theta \cos \varphi \right) U_1 / m \\
\dot{Y} & = \left( \cos \varphi \sin \psi - \sin \varphi \cos \theta \sin \psi \cos \varphi \right) U_1 / m \\
\dot{Z} & = -g + \left( \cos \theta \cos \varphi \sin \psi \right) U_1 / m \\
p & = \frac{I_{xy} - I_{zz}}{I_{xx}} q - \frac{J_{TP}}{I_{xx}} q + \frac{U_2}{m} \\
q & = \frac{I_{zz} - I_{xx}}{I_{yy}} p - \frac{J_{TP}}{I_{yy}} p + \frac{U_3}{m} \\
r & = \frac{I_{xx} - I_{yy}}{I_{zz}} p + \frac{U_4}{m} \\
U_1 & = T_1 + T_2 + T_3 + T_4 = b \left( \Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2 \right) \\
U_2 & = \mu T_4 - T_2 = b \left( \Omega_1^2 - \Omega_2^2 \right) \\
U_3 & = \mu T_3 - T_1 = b \left( \Omega_2^2 - \Omega_3^2 \right) \\
U_4 & = \mu \Omega_1 \Omega_2 + \mu \Omega_2 \Omega_3 + \mu \Omega_3 \Omega_4 \\
\end{align*}
\]

(1)

\(T_i\) and \(\Omega_i\) represent thrust and rotational velocity of \(i\)-th propeller respectively. As well as \(b\), \(d\) represent thrust and drag coefficient of propeller and also \(\mu\) addresses arm length of the system. Angular acceleration in body frame \((\dot{\rho}, \dot{q}, \dot{r})\) and inertial frame \((\dot{\rho}, \dot{\theta}, \dot{\psi})\) are not equal each other. For less variation of euler angles from zero position, it can be assumed that transmission function \((\Gamma_0)\) is close to identity matrix then: \((\dot{\phi}, \dot{\theta}, \dot{\psi}) = (\dot{p}, \dot{q}, \dot{r})\)

Motor model should be added to the quadrotor dynamics. So, the final form of the motor model is as [5]:

\[
\begin{align*}
\left( I_p + \eta N^2 J_M \right) \dot{w}_p & = -\frac{K_E K_M}{R} - \eta N^2 w_p - dw^2_p + \frac{K_M}{R} \eta N v \\
\end{align*}
\]

(3)

Based on the equation of energy consumption of brushless motors, the cost function is considered as:

\[
E_W = \int \left[ k_1 u^T - k_2 w_p^T u \right] dt
\]

(4)

\[
\left[ N_1, N_2, N_3 \right]^T = f_{16x1} \left( N_1, N_2, N_3, u \right)
\]

(5)

\[
N_{1,2,3} = \left[ w_p, X, Y, Z, \phi, \theta, \psi, X, Y, Z, \phi, \theta, \psi \right]
\]

(6)

\[
\left| u^T \right| \leq 1, \quad (i = 1, 2, 3, 4)
\]

(7)

Where \(k_1 = \frac{V^2}{R}, \quad k_2 = \frac{K \eta \eta \epsilon}{R} \), \(u \) and \(w_p \) are \(4 \times 1\) vectors. To extract the optimal control inputs which guarantee the minimum energy trajectory and satisfy the constraints, the Hamiltonian is constructed as:

\[
H = k_m u^T - k_2 w_p^T u + \frac{1}{2} \left[ f \left( N_1, N_2, N_3, u \right) \right]
\]

(8)

3. Laboratory prototype

To validate results and to compare it with experimental data, a prototype was designed and used. Sensors data and control commands are conveyed through an ATMega32A microcontroller with sample time 0.02 seconds. The produced thrust by propeller is measured according to Pulse-width modulation (PWM) signal applied to motor. Because of lack of encoder, the rotational velocity of propeller was determined based on mathematical model of motor-propeller. Extractive experimental data of motor has been presented in Table 1.

\[
\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
\text{PWM} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\hline
\text{312.5} & 312.5 & 312.5 & 312.5 & 312.5 & 312.5 & 312.5 & 312.5 \\
\hline
\text{315.5} & 315.5 & 315.5 & 315.5 & 315.5 & 315.5 & 315.5 & 315.5 \\
\hline
\text{317.5} & 317.5 & 317.5 & 317.5 & 317.5 & 317.5 & 317.5 & 317.5 \\
\hline
\text{319.5} & 319.5 & 319.5 & 319.5 & 319.5 & 319.5 & 319.5 & 319.5 \\
\hline
\end{array}
\]

Table 1. The results of experimental data of motor

4. Results and discussion

In order to extract consumption energy of minimum energy trajectory and evaluation and comparison with common Trapezoidal velocity profile, simulations were performed in different time and positional conditions which its results have been presented in Table 2. It illustrates that if the distance between start and end
points increase in constant duration flight, the amount of saving energy of optimal trajectory in comparison with Trapezoidal velocity profile will decrease.

### Table 2. The consumption energy in various condition

<table>
<thead>
<tr>
<th>$t$ (s)</th>
<th>$X_f$</th>
<th>$Y_f$</th>
<th>$Z_f$</th>
<th>Min energy (J)</th>
<th>Trapezoidal velocity profile (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>532.86</td>
<td>556.80</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>0</td>
<td>0</td>
<td>531.71</td>
<td>554.56</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>15</td>
<td>0</td>
<td>527.01</td>
<td>549.08</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0</td>
<td>20</td>
<td>537.78</td>
<td>558.44</td>
</tr>
<tr>
<td>20</td>
<td>25</td>
<td>0</td>
<td>0</td>
<td>1062.7</td>
<td>1097.26</td>
</tr>
<tr>
<td>20</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>1066.3</td>
<td>1102.69</td>
</tr>
</tbody>
</table>

Also, Table 2 shows that the difference of energy consumption between optimal and Trapezoidal profiles rise with prolonging duration flight. Results demonstrate that the amount of saving energy is almost 4%. Fig 1 shows the optimal energy consumption for one of a more used condition.

![Fig 1. The energy consumption profile along the minimum energy path](image1)

To validate simulation results and compare with actual status, the PWM signals corresponding with obtained voltages from simulation were applied to motor. The experimental data including motors voltages and currents is shown in Fig 2. The amount of energy consumption was determined using voltage and current experimental data which is about 304 J. It is compatible with simulation result which shows energy consumption 298 J.

![Fig 2. Voltage and current obtained from sensors](image2)

There is no explicit relation among flight parameters. Having a defined function which link these parameters to each other could be profitable. Using simulation results for a wide range of different times and distances and also using fitting methods an equation was derived as:

$$E(t, L) = 1063t + 19.38t^2 + 26.35t^3 - 19.13tL + 1761$$  \(9\)

Where $t$, $E$ and $L$ represent duration flight, energy consumption in optimal mode and distance between start and end points, respectively.

### 5. Conclusion

Results demonstrate that the most effective parameter in energy consumption of a quadrotor is duration flight. They show that relative difference between energy consumption of optimal and Trapezoidal profile decreases with increasing duration flight. Experimental results verify the validation of simulation results. Moreover, the presented mathematical relation reveals strict conformity with experimental data and therefore it can be used and cited for similar researches. Eventually in short time, the influence of optimal path is more while Trapezoidal velocity profile is recommended for middle and long time.

### 6. References


