Numerical Simulation of Filter Cake Formation on Borehole Walls Considering Effect of Non-Newtonian Drilling Fluid and Eccentricity

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ABSTRACT: In overbalanced drilling operation, due to the pressure difference between the drilling fluid and formation mud filtration happens in which a layer of cake develops on the sides of a wellbore wall. In this research a numerical procedure was used for modeling and simulation of the cake growth, in which at first, three-dimensional flow field of non-Newtonian drilling mud has been computed using finite difference scheme and using bipolar coordinate. Next, permeation flux of the filtrate is evaluated using the Darcy equation and the rate of cake growth has been calculated based on the probability of particle adhesion and using an explicit 4th order Runge-Kutta method. In the present paper, the effects of drill string rotation, eccentricity and particle radius on the cake thickness and permeate flux are studied. The results show filter cake thickness and filtrate flux remains invariable by increasing drill string rotation due to drilling fluid recirculation in the annulus. Increasing eccentricity not only changes the profile of filter cake thickness and filtrate flux but also increases the difference between its maxima and minima. Furthermore by decreasing particle size, the filter cake thickness increases and filtrate flux decreases.

1- Introduction

In the drilling process of oil wells, due to the pressure difference between the drilling fluid and formation, one can observe a mud filtration, which means the mud solids in the drilling mud sediment on the sides of the well, while they are penetrating into the formation. Therefore, a layer of mud cake gradually develops on the sides of a wellbore wall. The presence of mud cake on the sides of the formation is a beneficial occurrence, as it can reduce the amount of filtrate, which subsequently decreases the formation damage. On the other hand, thick filter cake features several demerits: decreasing the efficient well diameter, excessive torque when rotating the drill string, and excessive drag when pulling it. Williams [1] investigated cake filtration using a lab well model and showed that filtration rate, which is a function of pressure difference, mud flow and mud characteristics, reaches to a constant value after a short time. Prokop [2] conducted a laboratory experiment for both dynamic and static cases of radial filtration of drilling mud and showed that in dynamic case, filter cake formation depends not only upon filtration characteristics of the mud, but also upon the erosion action of the drilling fluid. Fisher et Al. [3] simulated the process of cake filtration considering eccentricity using a numerical model and revealed that as the eccentricity increases, the thickness profile of the cake loses its symmetry. In this study, a numerical method was used for modeling and simulation of the cake growth in which at first, 3D flow field of non-Newtonian drilling mud has been computed using finite difference scheme and using bipolar coordinate. Then, permeation flux of the filtrate is evaluated using the Darcy equation and the rate of cake growth has been calculated based on the probability of particle adhesion and using an explicit 4th order Runge-Kutta method.

2- Modeling

To prepare a model for cake formation process, three simultaneous phenomena should be formulated: (i) Free flow of non-Newtonian drilling fluid through eccentric annulus, (ii) cake formation due to deposition of solid part of drilling fluid on the borehole walls and (iii) porous flow of liquid part of the drilling fluid into the formation.

2-1- Modeling free flow in the annulus

Considering steady, incompressible, laminar, isothermal, and fully developed axial flow, one can write mass and momentum equations as follows [4]:

\[ \frac{\partial v_i}{\partial x_i} = 0 , \quad \frac{\partial v_j}{\partial x_j} = g_j - \frac{1}{\rho} \frac{\partial p}{\partial x_j} + \frac{1}{\rho} \frac{\partial \tau_{ij}}{\partial x_j} \]  (1)

where \( x_i \) and \( v_i \) are the position and velocity vectors, respectively. \( g_j \) is the gravitational acceleration vector and \( \tau_{ij} \) is the viscous shear stress tensor, which for a Power-law fluid is related to the strain rate tensor as:

\[ \tau_{ij} = K \dot{\gamma}_{ij}^p \]  (2)

To generate the computational grids for solving three-dimensional equations of motion considering eccentricity, a bipolar coordinate was used. A computational code was developed to solve the proposed model using finite difference scheme. For discretization, one can take advantage of second order central difference (with first order upwind method), as well as utilizing SOR approach for solving the equations. The complete solution and algorithm can be found in the literature.
2-2- Modeling filtrate flow through formation

The Darcy equation is used for modeling filtrate flow, considering filtrate as a Newtonian fluid, and radial flow, the velocity of permeating fluid can be obtained as:

\[
\left. u \right|_{r=R_o} = \frac{\Delta p}{R_o \mu} K_c K_f \frac{R_o}{R_c} \frac{R_o}{R_c - h} + K_c L_n \frac{R_o}{R_c} \ln \frac{R_c}{R_c - h}
\]

where \(K_c\) and \(K_f\) are permeability of the cake and formation respectively and \(\Delta p\) is the pressure drop.

2-3- Modeling cake formation on the wellbore wall

The model of cross-flow filtration, which is used in our modeling, is presented by Stamatakis and Tien [6].

3- Numerical Approach for Cake Formation

A numerical approach based on the finite difference method was used to simulate the cake growth in a drilling process. In which (as shown in Fig. 1), first, the free flow of drilling mud in the annulus is calculated to obtain velocity gradient on the wellbore wall, and then velocity of permeating fluid equation along with cake growth equation will be solved using an explicit 4th order Runge-Kutta method.

4- Results

Fig. 2 shows the rate of cake growth and permeate flux, which is high at the start of filtration and after about 2000 seconds, it reaches a constant value. Increasing cake thickness reduces the rate of fluid loss and therefore, the permeate flux decreases sharply at the start of filtration and reaches to a constant value, as the cake thickness reaches its steady state value.

The effect of eccentricity on the cake thickness is shown in Fig. 3. Increasing eccentricity not only changes the profile of filter cake thickness and filtrate flux but also increases the difference between its maxima and minima.

Increasing rotational speed leads to appearance and growth of flow recirculation in wide gap of the annulus which cause a thinner cake as shown in Fig. 4. Finally, as shown in Fig. 5, the cake thickness decreases by increasing particle radius.
5- Conclusions

1. Increasing eccentricity changes the profile of cake thickness and increases the difference between its maxima and minima.

2. By rotation of drill string, the cake thickness decreases and its extrema move to the narrowing and widening gap of the annulus.

3. Increasing drill string radius and reducing particle size lead to a thicker cake profile on the wall.

References
