**ABSTRACT:** In this research, natural convection of Al$_2$O$_3$-water nanofluid in a square cavity with top and bottom curved boundaries has been investigated using lattice Boltzmann method. A D2Q9 model has been used for both the hydrodynamic and thermal equations. Furthermore, in order to study the effect of nanoparticle size on the average Nusselt number, a two-component model has been used and two separate equations (one for each component) have been solved. Drag and buoyancy forces are included for coupling of these equations. Rayleigh number has been varied from $10^4$ to $10^6$ while volume fraction of nanoparticles is selected in a range of 0 to 0.05. Three nanoparticle sizes, namely 20, 50 and 100 nm, have been used in the simulations. Results show that the main factor for controlling the effectiveness of average Nusselt number of nanofluids compared to the base fluid is the volume fraction of nanoparticles. Results also reveal that nanoparticle size has a deteriorating effect on the Nusselt number enhancement of nanofluid. A correlation is then presented to predict the average Nusselt number of Al$_2$O$_3$-water nanofluid in the investigated criteria for Rayleigh number, volume fraction and size of nanoparticles.

1- Introduction

Nanofluids have been an interesting topic for researchers in recent years and different experimental, theoretical and numerical methods are presented from different researchers to explain behavior of such fluids. Among all numerical approaches the Lattice Boltzmann Method (LBM) has a strong capability to investigate nanofluids with great accuracy and speed. Hence, there are several reports on simulating nanofluids using LBM. Almost all papers published in the field of LBM simulation of nanofluids have utilized a single component model. There are two major shortcomings associated with this method. First, the method is inherently incapable of considering the effect of nanoparticle size. Second, in almost all published papers, the Maxwell-Garnett [1] correlation has been incorporated to the model for predicting thermal conductivity of nanofluids. It is shown that this correlation might lead to considerable errors [2].

Xuan and Yao [3] presented a two-component model in which four forces, namely buoyancy, drag, Brownian and potential, are used to couple two separate equations for each component through external forces. However, results presented by Qi et al. [4] showed that dependency of the results on the Brownian and potential forces are negligible. Application of lattice Boltzmann method was mostly done on geometries with Cartesian boundaries. However, efforts have been done in recent years in order to find second-order accurate methods of implementing boundary conditions on curved boundaries. Yan and Zu [5] have developed relations using Chapman-Enskog expansion of equations near curved walls. Their method is capable of applying both hydrodynamic and thermal boundary conditions with second order accuracy and hence, we have utilized it in our simulations.

In the present study, the effect of three main parameters, namely Rayleigh number, volume fraction of nanoparticles and nanoparticle diameter, on the average Nusselt number have been studied. The nanofluid is selected to be Al$_2$O$_3$-water with volume fraction of nanoparticles ranging from 0 to 5%. Rayleigh number is varied between $10^4$ to $10^6$ and three values (20, 50 and 100 nm) have been selected for nanoparticle sizes.

2- Numerical Method

The general form of discrete Boltzmann equation is as follows:

$$\mathbf{f}^e(x_i + e_i t, t + \tau) - \mathbf{f}^e(x_i, t) = Q + \mathbf{F}^e$$

where $Q$ represents the collision term and $\mathbf{F}^e$ is a representation for the effect of body forces on the LB equation for the $\sigma_{th}$ component.

The discrete velocity set, $e_i$, for a D2Q9 model with lattice speed equal to unity, i.e. $c = (\delta x/\delta t) = 1$, is as below:

$$e_i = \begin{cases} (0, 0), & i = 0 \[\sqrt{2}(\cos [(i - 1)\pi / 4], \sin [(i - 1)\pi / 4]), & i = 1, ..., 4 \end{cases}$$

$$\sqrt{2}(\cos [(2i - 9)\pi / 4], \sin [(2i - 9)\pi / 4]), & i = 5, ..., 8$$

The single relaxation form of the collision term results in the following:

$$\mathbf{f}^e(x_i + e_i t, t + \tau) = \mathbf{f}^e(x_i, t) - \frac{1}{\tau} [\mathbf{f}^e(x_i, t) - \mathbf{f}^e^e(x_i, t)]$$

where the superscript “eq” denotes the equilibrium distribution function for the local point.

**Keywords:**
- Natural convection
- Nanofluid
- Lattice Boltzmann method
- Average Nusselt number
- Curved boundary

M. Hosseini Abadshapoori, M. H. Saidi*
Mechanical Engineering, Center of Excellence in Energy Conversion (CEEC), Sharif University of Technology, Tehran, Iran

*Corresponding author, E-mail: saman@sharif.edu

Available Online: 13 August 2016
3- Results and Discussion

In order to investigate the effect of nanoparticle parameters, namely size of nanoparticles and volume fraction of nanoparticles on the Nusselt number, the following parameter is introduced based on the Nusselt number of nanofluids and pure fluid:

\[ Nu^* = \frac{Nu_{av} - Nu_{av, \phi=0}}{Nu_{av, \phi=0}} \]  

(4)

In other words, \( Nu^* \) is an indication of deviation of nanofluid Nusselt number from that of the respective pure fluid. It can be shown that \( Nu^* \) has dependency on at least three parameters, namely Rayleigh number, volume fraction of nanoparticles and nanoparticle diameter:

\[ Nu^* = f(Ra, \phi, d) \]  

(5)

Results show that \( Nu^* \) might be a function of Rayleigh number and nanoparticle diameter of the form \( Ra^{0.41} \phi^{0.175} \) where \( \varepsilon \) is defined based on the maximum diameter of nanoparticles which in the current simulations assumed to be 100 nm:

\[ \varepsilon = \frac{d}{d_{\text{max}}} \]  

(6)

Fig. 1 depicts variations of \( Nu^* \) in terms of this parameter for different volume fractions.

\[ \text{Figure 1. Variation of } Nu^* \text{ with nanoparticle size and Rayleigh number for different volume fraction of nanoparticles} \]

As illustrated in Fig. 1, \( Nu^* \) increases as we move to bigger values of which means that \( Nu^* \) increases with the increase of Rayleigh number and decrease of nanoparticle size. The opposing forces on particles increases with the increase of \( d \). To be specific, buoyancy and drag forces increase with the power of 3 and 1 of nanoparticle size, respectively. The enhancements of these forces result in a decrease on the rate of circulation of nanoparticles in the domain. This, in turn, decrease the rate of heat transferred between walls due to the presence of nanoparticles which cause a deterioration of \( Nu^* \). Furthermore, as can be seen in Fig. 1, \( Nu^* \) increases with the increase of volume fraction of nanoparticles. The difference between \( Nu^* \) of different volume fractions becomes more significant when the Rayleigh number increases. The dependency of the flow on the viscous forces reduces and rate of circulation of nanoparticles plays a more important role. Hence, presence of more nanoparticles results in a more significant rise of the \( Nu^* \).

Considering the effect of volume fraction of nanoparticles, Rayleigh number and nanoparticle size, the following relation can be introduced to predict the average Nusselt number of \( \text{Al}_2\text{O}_3 \)-water nanofluids:

\[ Nu_{av}^* = 0.219 Ra^{0.264} \times (1 + 0.057 Ra^{0.121} \varepsilon^{-0.175} \phi^{0.412}) \]  

(7)

The above equation is valid for \( 10^3 \leq Ra \leq 10^6 \), \( 0 \leq \phi \leq 0.05 \) and \( 0.2 \leq \varepsilon \leq 1 \). The maximum error of average Nusselt number prediction by the above equation is 8.2.

4- Conclusions

In the present study, we present a simulation of natural convection of \( \text{Al}_2\text{O}_3 \)-water nanofluid in a curved geometry based on a two-component model of lattice Boltzmann. Concluding remarks are as follows:

- The main parameter affecting \( Nu^* \) is the volume fraction of nanoparticles. Addition of nanoparticles to fluids enhances the Nusselt number. This increase might be explained through higher thermal conductivity of nanoparticles compared to that of the base fluids. However, addition of nanoparticles also increases viscosity of nanofluid which reduces the effective Rayleigh number. This effect is more significant at higher concentration of nanoparticles.
- \( Nu^* \) decreases as the diameter of nanoparticles increases due to the increase of external forces exerted on the nanoparticles.
- A correlation is provided to predict the average Nusselt number of \( \text{Al}_2\text{O}_3 \)-water nanofluid considering the effect of Rayleigh number, volume fraction of nanoparticles and main conclusions of the paper must be put here.

Acknowledgment

The authors sincerely thank the Iran National Science Foundation for their financial support through Project No. 940017 during the course of this work.

References

