Free and Forceld Whirling Analyses of Rotors with Multiple Unbalanced Discs Under Axial Force

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ABSTRACT: In this paper the set of equation for free and forced whirling analyses of rotors with any number of discs is derived. By considering gyroscopic effects, the rotor is modeled based on the Timoshenko beam theory and discs are considered as concentrated elements having both translational and rotational inertias. At the position of each disc, the rotor is imposed to distributed and concentrated axial forces which vary versus time. Also, transverse load composed of unbalanced masses and total weight of the system is considered. For a simply supported rotor, the free whirling analysis is investigated using Galerkin method and using Galerkin and Newmark-beta methods, the forced whirling analysis is studied numerically. Forward and backward frequencies and Campbell diagrams are presented in free whirling analysis and variation of deflection, bending moment and shear force in any point of the rotor are depicted versus time in forced whirling analysis. The most advantages of the presented paper are consideration of time-dependency of rotating speed in forced whirling analysis and its applicability for rotors with any number of mounted discs.

1- Introduction

2- Governing Equations
As depicted in Fig. 1, a uniform rotor of length L and diameter d rotating at angular velocity Ω with various numbers of discs is considered. The rotational speed increases from zero to its nominal value (Ω₀) in time t₀, as

\[ \Omega = \begin{cases} \frac{2 L}{t_0} \left( \frac{t}{t_0} \right)^2 & t \leq t_0 \\ 1 & t \geq t_0 \end{cases} \]  \tag{1}

Also, all discs are imposed on a distributed time dependent axial force (q(t)) and a concentrated one (F(t)). Using Newton’s second law, the set of governing equations can be written as

\[ kGA \left( \frac{\partial^2 u_x}{\partial z^2} + \frac{\partial^2 \varphi}{\partial z^2} \right) - R_0 + \sum_{i=1}^{N} P_i(t) H(z - z_i) \frac{\partial^2 u_x}{\partial z^2} - \sum_{i=1}^{N} P_i(t) \delta(z - z_i) \frac{\partial^2 u_x}{\partial z^2} + f_x(x, t) = \rho A + \sum_{i=1}^{N} M_i \delta(z - z_i) \frac{\partial^2 u_x}{\partial t^2} \]

\[ kGA \left( \frac{\partial^2 u_y}{\partial z^2} + \frac{\partial^2 \varphi}{\partial z^2} \right) - R_0 + \sum_{i=1}^{N} P_i(t) H(z - z_i) \frac{\partial^2 u_y}{\partial z^2} \]
Using Eq. (3) and the following complex variables created by gravity and unbalance masses as of elasticity and shear modulus, respectively and

\[ u \left( x, t \right) = -\rho A g \sum_{i=1}^{N} M_i g \delta (z - z_i) \]

\[ + \Omega^2 \sum_{i=1}^{N} m_i^f e_i \left[ \cos \left( \Omega t + \theta_i \right) \right] \delta (z - z_i) \]

Using Eq. (3) and the following complex variables

\[ u = u_s + j u_f, \quad \varphi = \varphi_s + j \varphi_f \]

\[ f = f_s + j f_f, \]

the set of governing equations can be written as

\[-\rho A + \sum_{i=1}^{N} M_i \delta (z - z_i) \left( \frac{\partial^2 u}{\partial t^2} + k G A \left( \frac{\partial^2 u}{\partial z^2} + j \frac{\partial \varphi}{\partial z} \right) \right) \]

\[-R_g + \sum_{i=1}^{N} P_i \left( t \right) H \left( z - z_i \right) \left( \frac{\partial^2 u}{\partial t^2} - \sum_{i=1}^{N} P_i \left( t \right) \delta (z - z_i) \right) \]

\[= \rho A g - j \Omega^2 \sum_{i=1}^{N} m_i^f e_i \sin \left( \Omega t + \theta_i \right) \delta (z - z_i) \]

\[= \sum_{i=1}^{N} \left[ M_i g - m_i^f e_i \Omega^2 \cos \left( \Omega t + \theta_i \right) \right] \delta (z - z_i) \]

\[-\rho I + \sum_{i=1}^{N} I_i \delta (z - z_i) \left( \frac{\partial^2 \varphi}{\partial t^2} + 2j \frac{\partial^2 \varphi}{\partial z^2} + \rho I + \sum_{i=1}^{N} I_i \delta (z - z_i) \right. \]

\[+ EI \frac{\partial^2 \varphi}{\partial z^2} \]

\[= \sum_{i=1}^{N} F_i e^{j \omega_i \delta (z - z_i)} \]

Inserting Eq. (7) into Eq. (5) and using the Galerkin method, the set of governing equations can be written as

\[ [M] [\ddot{X}(t)] + [G] [\dot{X}(t)] + [K] [X(t)] = [F(t)] \]

(8)

Eq. (8) can be solved using Newmark-beta method [8] and for free vibration analysis, the following relation can be used:

\[ \omega^2 [M] + \omega [C] + [K] = 0 \]

(9)

4- Results and Discussion

In order to investigate the effect of axial forces on Campbell diagrams, a rotor of length \( L = 4 \) m and diameter \( d = 30 \) mm which carries a single disc of diameter \( d = 60 \) mm and thickness \( t = 15 \) mm is employed. The disc is located at \( z = L/3 \) and is imposed on a distributed axial force of intensity \( q \). For the first two modes, the effect of \( q \) on the Campbell diagrams is depicted in Figs. 2 and 3. These figures show that both the decrease and increase in frequencies can be found. Actually, external axial forces create tension or compression in different parts of the rotor and therefore its stiffness increases in some parts and decreases in other parts which leads to a rise in some frequencies and the decrease in other ones.

Consider a rotor of length \( L = 5 \) m, diameter \( d = 60 \) mm with three unbalanced discs of the properties presented in Table 1. The rotating speeds increases in two seconds from zero to its nominal value \( (\Omega = 100 \text{ rpm}) \). Figs. 4 to 7 show the variation of components of displacement and bending moment in the middle length of the rotor. These figures show that

![Figure 2. Campbell diagram for the first mode](image2)

![Figure 3. Campbell diagram for the second mode](image3)
displacement and bending moment vary with time between their maximum and minimum values; these values can be used directly to study fatigue in the rotor.

### 5- Conclusions

Using Galerkin and Newmark-beta methods, free and forced whirling analyses of rotors with any number of discs under time variable axial forces were presented. The numerical results showed that Campbell diagram of the rotor was strongly affected by axial forces and both the decrease and increase in frequencies can occur. Also, it was concluded that bending moment vary periodically with time between its lower and upper bounds; these values are very useful and necessary to study fatigue of the rotor.

### References


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