



Free Vibration of, Functionally graded materials Cylindrical Shells on Elastic Foundation under Axial force, Lateral Pressure and Different Boundary Conditions

A. Hadi¹, S. Shakhesi^{2*}, H. Ovesy³, J. Fazilati¹

¹ Aerospace Research Institute, Ministry of Science, Research and Technology, Tehran, Iran

² Space Transportation Research Institute, Iranian Space Research Center, Tehran, Iran

³ Aerospace Engineering Department, Amirkabir University of Technology, Tehran, Iran

ABSTRACT: Free vibration characteristics of functionally graded materials cylindrical shells surrounded by elastic medium under axial force, lateral pressure and different boundary conditions using wave propagation method are investigated in this paper. The material properties of functionally graded materials are assumed to be graded in the thickness direction according to the power law. The elastic medium is assumed as two-parameter Pasternak elastic foundation. Governing equations based on the first order shear deformation theory of Sanders-Koiter for the cylindrical shell resting on elastic foundation under mechanical loads are derived by using Hamilton's principle. By assuming displacement field in wave propagation form, governing equations are solved. Natural frequencies of cylindrical shell under various boundary conditions are obtained and compared with the results in the literature. It is seen that using displacement field in wave propagation form, acts as an effective and reliable method and gives the acceptable results for various boundary conditions. Although it is shown that for different boundary conditions and geometry dimensions, accuracy of the wave propagation approach is different. In addition, based on the developed theory the effects of different boundary conditions, axial force, lateral pressure and elastic foundation parameters on vibration behavior of functionally graded cylindrical shell are investigated.

Review History:

Received: 6 January 2017

Revised: 17 March 2017

Accepted: 7 May 2017

Available Online: 17 May 2017

Keywords:

Free vibration

Functionally graded materials

Cylindrical shells

Axial force and lateral pressure

Elastic foundation

1- Introduction

Shells are one of the most important and applicable structural elements in various fields of engineering and technology fields such as aerospace, chimney design, pipe flow and so on. Widespread use of cylindrical shells is the reason for significant growth of studies in this area. The study of the free vibrations of shells has been carried out extensively by Leissa [1]. He used beam functions to consider different boundary conditions. The effects of different boundary conditions on the vibration characteristics of the cylindrical shells are studied by some researchers [2-5]. In most of the mentioned cases, the solution methods to consider different boundary conditions are often complicated and may have limitation. Recently, a new method for describing displacement field in wave propagation form to predict natural frequencies of cylindrical shells is developed by Zhang et al. [6]. Some advantages of this method can be summarized as simple application, capability to develop the method to different geometry and boundary conditions and high processing speed. None of the previous studies is specifically performed for investigation of the effectiveness and accuracy of the wave propagation method in free vibration analysis of Functionally Graded (FG) cylindrical shells and application of this method to study the effects of elastic foundation, axial force and lateral pressure and different boundary conditions on the vibration behavior of the FG cylindrical shells.

Therefore, this study aims at this two mentioned goals. To do this, based on the first order shear deformation shell theory of Sanders-Koiter, a set of differential equations of motion is derived in terms of the shell middle surface displacement

components. The elastic medium is assumed as two-parameter Pasternak elastic foundation. Governing equations are solved by employing wave propagation approach. Results are presented on the frequency characteristics and influence of elastic foundation, mechanical loads and different boundary conditions. The present analysis is validated by comparing results with those available in the literature.

2- Methodology

The present study considers functionally graded material composed of two materials in which material properties are varied along the shell thickness according to the power law. Considering elastic foundation, shear deformation and rotary inertia effects, governing equations of cylindrical shell under axial force and lateral pressure based on Sanders-Koiter kinematics relations are derived in terms of the middle surface displacement components.

The displacement of the shell can be expressed in the form of wave propagation, associated with axial modal parameter k_m and circumferential wave number n , and defined by:

$$\begin{aligned} u &= A \cos(n\theta) e^{i(\omega t - k_m x)} \\ v &= B \sin(n\theta) e^{i(\omega t - k_m x)} \\ w &= C \cos(n\theta) e^{i(\omega t - k_m x)} \\ \psi_x &= D \cos(n\theta) e^{i(\omega t - k_m x)} \\ \psi_\theta &= E \sin(n\theta) e^{i(\omega t - k_m x)} \end{aligned} \quad (1)$$

where A , B , C , D and E are the wave amplitudes. ω is the circular driving frequency and $i = \sqrt{-1}$. In order to calculate the natural frequencies, it is only necessary to determine the modal parameter k_m in the axial direction that strongly

Corresponding author, E-mail: s.shakhesi@isrc.ac.ir

depends on the boundary conditions. However, if one is only interested in flexural vibration, one may use the beam function to determine the modal parameter of cylindrical shells in the axial direction by assuming the flexural mode shapes of cylindrical shells in the axial direction to be of the same form as that of a transverse vibration beam of the same boundary conditions [6]. Therefore, with regard to boundary condition, suitable axial modal parameter is chosen and by substituting displacement field in form of wave propagation in the governing equations, a set of linear equations are obtained. These equations are solved by imposing the condition of non-trivial solutions and equating the characteristic determinant of the coefficient matrix to zero.

3- Results and Discussion

To check the effectiveness and accuracy of the wave propagation approach, the results obtained are compared with those in the literature for different boundary condition. Considered boundary conditions are consist of simply Supported-Simply Supported (SS-SS), Clamped-Clamped (C-C), Clamped-Simply Supported (C-SS) and Clamped-Free (C-F). It should be noted that in some cases finite element model of the cylindrical shell is prepared by authors in Nastran commercial package for comparison. Also, validation of the present development for the cases of FG cylindrical shells and elastic foundation is performed. Based on the validated theory, the effects of different boundary conditions, elastic foundation, axial force and lateral pressure on the vibration behavior of FG cylindrical shells are investigated. Variation of natural frequencies with circumferential wave number for a FG cylindrical shell with volume fraction exponent $N=5$ and elastic foundation with different Winkler and shear parameters is studied. Axial wave number $m=1$ is chosen in calculation. It is observed that for the shell in interaction with elastic foundation by increasing foundation parameters, the effect of different boundary conditions is only for the case of circumferential wave number $n=1$. In the other words, increasing circumferential wave number has no effect on natural frequencies for different boundary conditions. Moreover, by examining different greater values of elastic foundation parameters, it is shown that there is a little change in natural frequencies. In other words, it can be stated that adding elastic foundation with suitable parameters to the shell, would result decreasing the effects of different boundary conditions on free vibration behavior.

Next study is concerned with variation of natural frequencies with Winkler parameter for FG cylindrical shell with volume fraction exponent $N=5$, axial and circumferential wave numbers $m=n=1$ for two values of shear parameter. The results are shown in Fig.1. As it is seen, by increasing Winkler parameter, natural frequencies are significantly increased at first then by more increasing the parameter there is a little change in natural frequencies. Moreover, for each value of shear parameter, there is a specified value of Winkler parameter in which the effect of boundary conditions is changed. In other words, for less than this value of Winkler parameter natural frequencies are decreased for the case of C-F, SS-SS, C-SS and C-C, respectively and vice versa. In addition, it is seen that by increasing the shear parameter this specific Winkler parameter (location of changing the effect of boundary conditions) is shifted to less value (Fig. 1 (b)).

4- Conclusion

This research is involved with free vibration analysis of FG

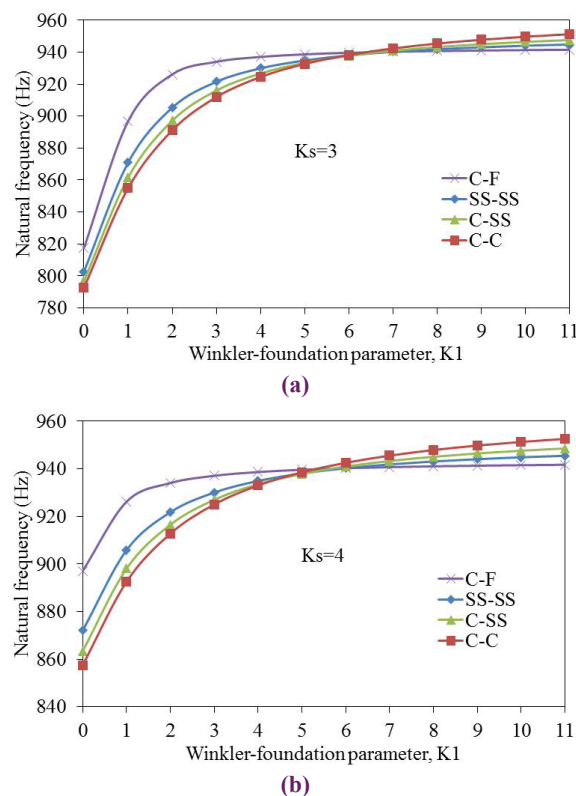


Fig. 1. Natural frequency variation of FG cylindrical shell with Winkler parameter for (a) $K_s=3$ (b) $K_s=4$

cylindrical shells surrounded by elastic medium under axial force, lateral pressure and different boundary conditions using wave propagation method. Some of important achievements of this research are listed below:

- The wave propagation approach acts as an effective and reliable method in free vibration study of cylindrical shells under different boundary conditions. The method can be extended to analyze the vibration of shells with more complex boundary conditions.
- It is observed that for a FG cylindrical shell under elastic medium, the effect of boundary conditions is only limited to lower values of circumferential wave number n . Therefore, adding elastic foundation with suitable parameters to the shell would result decreasing the effects of boundary conditions on free vibration behavior.
- It is observed that natural frequencies of the FG cylindrical shell are significantly increased at first when Winkler parameter increases then by more increasing the parameter there is a little change in natural frequencies. Moreover, for each value of shear parameter, there is a specific value of Winkler parameter in which the effect of boundary conditions is changed.
- Increasing tensile axial force and lateral pressure leads to increase natural frequencies. Moreover, for the shell under lateral pressure the effects of different boundary conditions is only for the case of $n=1$.

References

- [1] A.W. Leissa, Vibration of Shells, The Acoustical Society of America, 1973.
- [2] K. Forsberg, Influence of boundary conditions on modal characteristics of cylindrical shells, Journal of American institute of aeronautics and astronautics, 2 (1964) 182-189.

- [3] C.B. Sharma, Vibration characteristics of thin circular cylinders, *Journal of sound and vibration*, 63 (1979) 581-592.
- [4] A. Messina, K.P. Soldatos, Ritz-type dynamic analysis of cross-ply laminated circular cylinders subjected to different boundary conditions, *Journal of Sound and Vibration*, 227(4) (1999) 749-768.
- [5] R. Ansari, M. Darvizeh, M. Hemmatnezhad, Vibration Analysis of FGM Cylindrical Shells Under Various Boundary Conditions, *Iranian Aerospace Society*, 5(3) (2008) 129-138.
- [6] X.M. Zhang, G.R. Liu, K.Y. Lam, Vibration analysis of thin cylindrical shells using the wave propagation approach, *Journal of sound and vibration*, 239(3) (2001) 397-403.

Please cite this article using:

A. Hadi, S. Shakhesi, H. Ovesy, J. Fazilati, Free Vibration of, Functionally graded materials Cylindrical Shells on Elastic Foundation under Axial force, Lateral Pressure and Different Boundary Conditions, *Amirkabir J. Mech. Eng.*, 50(5) (2018) 357-360.

DOI: 10.22060/mej.2017.12362.5319



