



Free Vibration Analysis of Two-Dimensional Functionally Graded Annular Sector Plates with Piezoelectric Layers Resting on Two-Parameter Elastic Foundation

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ABSTRACT: In this paper, free vibration analysis of two-dimensional functionally graded annular sector plates with two piezoelectric layers on elastic foundation using the Rayleigh-Ritz method is investigated applying the third order shear deformation theory of plates. Despite the existence of more complex equations, the third-order shear deformation theory of plates leads to more accurate results, this is due to the effects of transverse shear deformation and inertial rotation. The material properties of the annular plate is graded two-dimensionally in both of the circumferential and thickness directions. The volume fraction power law distribution is used to model the two dimensional functionally graded plate properties. The sinusoidal function is used to determine the electrical potential of the piezoelectric layers. The assumed potential function satisfies the Maxwell electrostatic equations. To obtain the natural frequencies of the plate, the Lagrangian function is defined by subtracting the kinetic and potential energies of the plate and then the Hamilton's principle and the Ritz method is applied to obtain the natural frequencies of the plate. Finally, the results are validated by various references and the effect of various parameters including the sector angle, ratio of thickness to radius of the plate and foundation coefficients on the natural frequency of the structure has been discussed.

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1- Introduction

Application of materials which their mechanical properties are graded two-dimensionally may improve design of structures which are subjected to two stress distributions along the two different directions. Consequently, dynamic behavior of structures may be affected according to two dimensional grading of the materials and may be improved by applying the piezoelectric layers. On the other hand, study of natural frequencies of plates is highly dependent on the plate theory which is applied to obtain the governing equations of the plate. While, natural frequencies of piezoelectric coupled annular/circular plates have been studied previously by Hosseini-Hashemi et al. [1], Duan et al. [2] and Liu et al. [3], less works has been done on free vibration analysis of thick annular sector plates made of functionally graded materials using higher order shear deformation theories of plates. In the present work, natural frequencies of thick plates made of two-dimensional functionally graded materials resting on Pasternak foundation are studied using the Rayleigh-Ritz method.

2- Methodology

2- 1- Dynamical equations of plate

In this study the properties of the plate is graded through thickness and circumferential directions so that the lower surface of $z = -h$ is fully ceramic. The first type of ceramics is located at $\theta = 0$, $z = -h$ and it is graded through the circumferential direction while the percentage of the first type ceramic is gradually reduced. The percentage of second type ceramic is added so that the pure second type ceramic is located at $\theta = \alpha$, $z = -h$. The upper surface at $z = h$ is fully

metal. The material which is located at position of $\theta = 0, z = h$ is first type metal entirely and it is graded circumferentially the same as ceramic part of the material. The following mathematical relationship are used to model the material properties of the plate [4].

$$E(\theta, z) = E_{m1}V_{m1} + E_{m2}V_{m2} + E_{c1}V_{c1} + E_{c2}V_{c2} \quad (1)$$

$$\rho(\theta, z) = \rho_{m1}V_{m1} + \rho_{m2}V_{m2} + \rho_{c1}V_{c1} + \rho_{c2}V_{c2} \quad (2)$$

Where, the values of V represents the volume fractions, E and ρ are modulus of elasticity and density of plate, the subscripts m_1, m_2, c_1, c_2 represent metal types I and II and ceramic types I and II. Values of β_θ and β_z are gradient indexes in circumferential and thickness directions respectively. Assuming the third order shear deformation theory of plates in cylindrical coordinate system, the displacement fields are as follows:

$$u(r, \theta, z, t) = z\psi_r(r, \theta, t) - \frac{4}{3h^2}(z)^3 \left(\psi_r + \frac{\partial w}{\partial r} \right) \quad (1)$$

$$v(r, \theta, z, t) = z\psi_\theta(r, \theta, t) - \frac{4}{3h^2}(z)^3 \left(\psi_\theta + \frac{\partial w}{r\partial\theta} \right) \quad (2)$$

$$w(r, \theta, z, t) = w_0(r, \theta, t) \quad (3)$$

According to the generalized Hooke's law, the stress-strain relations of the plate are

$$\begin{pmatrix} \sigma_{rr}^h \\ \sigma_{\theta\theta}^h \\ \sigma_{r\theta}^h \\ \sigma_{rz}^h \\ \sigma_{\theta z}^h \end{pmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{11} & 0 & 0 & 0 \\ 0 & 0 & Q_{33} & 0 & 0 \\ 0 & 0 & 0 & Q_{33} & 0 \\ 0 & 0 & 0 & 0 & Q_{33} \end{bmatrix} \begin{pmatrix} \varepsilon_{rr} \\ \varepsilon_{\theta\theta} \\ \varepsilon_{r\theta} \\ \varepsilon_{rz} \\ \varepsilon_{\theta z} \end{pmatrix} \quad (4)$$

where, $Q_{11} = \frac{E(\theta, z)}{1-\nu^2}$, $Q_{12} = \nu Q_{11}$ and $Q_{33} = \frac{E(\theta, z)}{2(1+\nu)}$. The parameter $E(\theta, z)$ is the modulus of elasticity as a function of θ and z , and ν is Poisson's ratio of the plate. Superscript h indicates the functional properties of the plate. The stress distribution in the piezoelectric layers are:

$$\begin{pmatrix} \sigma_{rr}^p \\ \sigma_{\theta\theta}^p \\ \sigma_{r\theta}^p \\ \sigma_{rz}^p \\ \sigma_{\theta z}^p \end{pmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{21} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & C_{55} \end{bmatrix} \begin{pmatrix} \varepsilon_{rr} \\ \varepsilon_{\theta\theta} \\ \varepsilon_{r\theta} \\ \varepsilon_{rz} \\ \varepsilon_{\theta z} \end{pmatrix} \quad (5)$$

$$- \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{31} \\ 0 & 0 & 0 \\ e_{15} & 0 & 0 \\ 0 & e_{15} & 0 \end{bmatrix} \begin{pmatrix} E_r \\ E_\theta \\ E_z \end{pmatrix}$$

wherein, E_r, E_θ, E_z are electric field intensity in the radial, tangential and transverse directions, respectively. Parameters $C_{11}, C_{12}, C_{21}, C_{22}, C_{44}, C_{55}$ are elastic properties of piezoelectric material and e_{31}, e_{15} are piezoelectric constants. Superscript indicates the properties of piezoelectric layers. A sine function which can satisfy the Maxwell's equation is used in order to model the potential function.

2- 2- Rayleigh-Ritz method

In Rayleigh-Ritz method, the unknown functions are substituted with an approximate function. Then, parameters of the approximated function are obtained by extremizing energy functional.

The energy functional is:

$$\Pi = U_{\max} - T_{\max} \quad (7)$$

In the above equation, U_{\max}, T_{\max} are function of the unknown parameters, a_{ij}, b_{ij}, c_{ij} . Derivatives of the energy functional with respect to unknown parameters should be vanished to extremize the energy functional.

3- Results and Discussion

In Table 1, the first natural frequency of two-dimensional functionally graded annular sector plates with piezoelectric layers resting on elastic foundation for 30, 45 and 60 sector angles is represented. In this table $\zeta = 0.16, \eta = 0.1, \delta = 0.04$ and the boundary conditions on all edges of the plate are simply supported. Fig. 1 represents first natural frequency of the two-dimensional functionally graded annular sector plate with piezoelectric layers resting on elastic foundation versus the sector angle for various foundation parameters. The material gradient power indexes are $(\beta_z, \beta_\theta) = (1, 1)$, and other parameters are $\zeta = 0.16, \eta = 0.1, \delta = 0.004$. In Fig. 2, the first natural frequency of two-dimensional functionally graded annular sector plates with piezoelectric layers resting on elastic foundation versus the thickness of graded layer is shown for various foundation parameters. The parameters which are used to obtain the results for this figure are

$$(\beta_z, \beta_\theta) = (2, 2), (K_w, K_s) = (50M N/m^2, 50M N/m^2).$$

Table 1: The first natural frequencies of the two-dimensional FG annular sector plate with piezoelectric layers resting on elastic foundation in rad/s ($\delta = 0.04, \zeta = 0.16, \eta = 0.1$)

$\alpha(\text{deg})$			(β_z, β_θ)	K_w, K_s (MN/m ²)
60	45	30		
12251.5	15518.6	22052.7	(0,0)	(0,0)
13635.2	17017.4	2376	(0,0)	(25,10)
14893.9	18365.4	25369.3	(0,0)	(50,20)
16111	20258	28469.6	(0,0)	(20,50)
10514.8	17017.4	18935	(0,0)	(0,0)
11818.1	14734.4	20555.9	(0.5,0)	(25,10)
12991.4	15992.7	22.58	(0.5,0)	(50,20)
14199.6	17748.7	24936.9	(0.5,0)	(20,50)
9635.6	12235.8	17337.6	(0,0)	(0,0)
10901	13573.2	18914	(1,0)	(25,10)
12034.1	14790.2	20368.8	(1,0)	(50,20)
13199.7	16481.8	23144.7	(1,0)	(20,50)
7971.2	10122.2	14342.7	(0,0)	(0,0)
10667.7	13008.4	17783.2	(0,1)	(25,10)
12808.6	15361.5	20658.4	(0,1)	(50,20)
14726.7	18392.6	25698	(0,1)	(20,50)

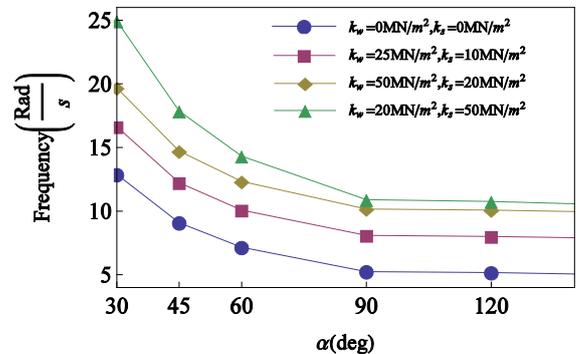


Fig. 1: The first natural frequency of two-dimensional FG annular sector plate versus the sector angle

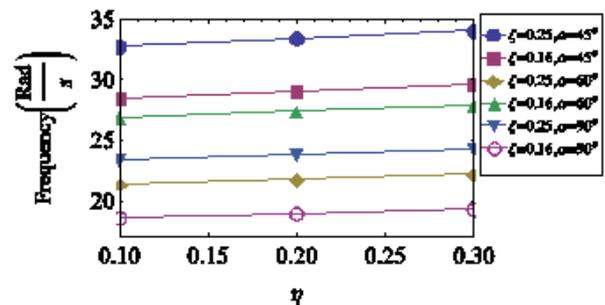


Fig. 2: The first natural frequency of two-dimensional FG annular sector plate versus the thickness of FG layer

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