



## Numerical Study of Brinkman Number Effects on Heated Viscoelastic Fluid Flow in Channel with Sudden Expansion

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**ABSTRACT:** In this paper, the inertial and non-isothermal flow of viscoelastic fluid inside the symmetric planar sudden expansion channel with an expansion ratio of 1:3 has been numerically investigated in the range of Brinkman numbers ( $0.01 \leq Br \leq 20$ ). The rheological and nonlinear model of Phan Thien-Tanner (PTT) is used for modeling viscoelastic fluid behavior. The finite volume method (FVM) is employed to discretize the governing equations and the PISO algorithm is used to solve these equations simultaneously. Due to the significant effect of temperature changes on the viscoelastic fluid properties, these properties are considered as temperature-dependent and the viscous dissipations term is considered in the energy equation. The main purpose of this study is to investigate the effects of Brinkman numbers on the heat generation by viscous dissipations term used in the energy equation. Therefore, the streamlines, the length of vortices, the isothermal lines, the distributions of velocity and temperature and local Nusselt numbers have been examined in the channel expanded part. The results show that for the hydrodynamic and thermally developing zone, the maximum value of the local Nusselt numbers on the walls of the channel expanded part is located at the end of the first and second vortices.

### 1- Introduction

Due to the complicated nature of non-Newtonian fluids (in contrast to the Newtonian fluids), they show specific behaviors by changes in the stream and temperature, especially in the channels with sudden and gradual expansion. Hence, investigation of viscoelastic fluid behavior for non-isothermal flow in these cases is valuable and important. In previous studies [1-3], researchers have been numerically simulated the creeping flow of polymer melt in the planar channel with asymmetric sudden expansion. They used the models of the Cross and modified power-law and considered the temperature-dependency for viscosity and relaxation time to investigate the temperature distribution. Most previous studies are related to the heat transfer of viscoelastic fluid flow in asymmetric sudden expansion with small values of Weissenberg number, Reynolds number and the low temperature difference between the inlet and the walls of the channel. The innovation of the present study is the use of the nonlinear form of Phan Thien-Tanner (PTT) model for simulating the heat transfer of non-creeping flow of viscoelastic fluid in the planar channel with symmetric sudden expansion which the fluid properties are considered temperature-dependent. A schematic plan of channel geometry has been shown in Fig. 1. In this figure, the height and length of the first and second sections of the channel are named  $h$ ,  $L_1$ ,  $H$  and  $L_2$  respectively. Also, simulation of this problem has been done using Finite Volume Method (FVM) and PISO

algorithm in the open source software of OpenFOAM.

### 2- Governing Equation and Solution Method

The governing equations including the continuity, momentum, and energy equations are as follows [4]:

$$\nabla \cdot \mathbf{U} = 0 \quad (1)$$

$$\nabla \cdot (\rho \mathbf{U} \mathbf{U}) = -\nabla p + \nabla \cdot \boldsymbol{\tau}_s + \nabla \cdot \boldsymbol{\tau}_p \quad (2)$$

$$\nabla \cdot (\rho c_p(T) \mathbf{U} T) = \nabla \cdot (k_f(T) \nabla T) + [\boldsymbol{\tau}_s + \boldsymbol{\tau}_p] : \mathbf{D} \quad (3)$$

$$\mathbf{D} = \frac{1}{2} [\nabla \mathbf{U} + \nabla \mathbf{U}^T] \quad (4)$$

$$\boldsymbol{\tau}_s = \eta_s(T) [\nabla \mathbf{U} + \nabla \mathbf{U}^T] \quad (5)$$

Variables of  $U$ ,  $p$ ,  $T$ ,  $\rho$ ,  $c_p(T)$  and  $k_f(T)$  are velocity vector, pressure, temperature, density, specific heat capacity and thermal conductivity respectively. Also  $\mathbf{D}$  and  $\eta_s(T)$  represent the deformation tensor and viscosity of Newtonian solvent respectively. The total stress tensor is obtained from the

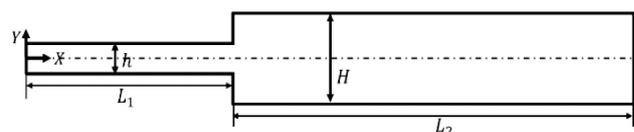


Fig. 1. Schematic plan of the sudden divergent channel

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**Table 1. The values of constants used in the current study [7]**

$k_s^*=0.00118$ (1/°C)	$\alpha=1720$ (K)	$\rho=1226$ (kg/m <sup>3</sup> )
$C_{p,s}^*=-0.00112$ (1/°C)	$T_i=190$ (°C)	$T_w=290$ (°C)
$\eta_0(T_i)=4.07$ (Pa.s)	$k_0^*=0.7753$	$U_i=0.83$ (m/s)
$\lambda(T_i)=2.4096$ (s)	$T_0=190$ (°C)	$C_{p,0}^*=1.2122$

summation of the Newtonian stress ( $\tau_s$ ) and the polymeric stress ( $\tau_p$ ). The polymeric stress is calculated by the exponential form of Phan Thien-Tanner model as follows [5,6]:

$$F(\text{tr}\tau_p)\tau_p + \lambda(T) \left[ \nabla \cdot (\mathbf{U}\tau_p) - \tau_p \cdot \nabla \mathbf{U} - (\nabla \mathbf{U})^T \cdot \tau_p \right] + \xi \left[ \tau_p \mathbf{D} + \mathbf{D}\tau_p \right] = \eta_p(T) \left[ \nabla \mathbf{U} + \nabla \mathbf{U}^T \right] \quad (6)$$

$$F(\text{tr}\tau_p) = \exp \left( \frac{\varepsilon \lambda(T)}{\eta_p(T)} \text{tr}\tau_p \right) \quad (7)$$

The temperature-dependent parameters  $\lambda(T)$  and  $\eta_p(T)$  are relaxation time and polymeric viscosity respectively, and the values of adjustable parameters are  $\varepsilon=0.02$  and  $\xi=0.04$ . Also, the temperature-dependency properties of the fluid are defined as follows [7]:

$$\eta_0(T) = \eta_0(T_i) \times a(T), \quad k_f(T) = k_f(T_i) \times [k_0^* + k_s^* T] \quad (8)$$

$$\lambda(T) = \lambda(T_i) \times a(T), \quad c_p(T) = c_p(T_i) \times [c_{p,0}^* + c_{p,s}^* T]$$

$$a(T) = \exp \left[ \alpha \left( \frac{1}{T+273.15} - \frac{1}{T_0+273.15} \right) \right] \quad (9)$$

Parameters  $\eta_0(T_i)$ ,  $\lambda(T_i)$ ,  $k_f(T_i)$ , and  $c_p(T_i)$  indicate the total viscosity, relaxation time, thermal conductivity and specific heat capacity at the inlet temperature of the channel. The values of these parameters are obtained by the dimensionless numbers and Table 1.

At the channel inlet, velocity and temperature are constant and uniform and at the channel output, the values of relative pressure and temperature gradient are equal to zero. For discretizing the convective term in the governing equations, the linear-upwind differencing scheme is used and the gradient and Laplacian terms are discretized by the central difference. After applying the boundary conditions, the components of velocity and mass flux is calculated using linear interpolation on the cells face of the grid. Also, the pressure equation is calculated on the face of grid cells and by using the new pressure, the velocity components are corrected. Then, the equation of polymeric stress is solved and after updating the temperature-dependent fluid properties, the energy equation is solved. This process is repeated as long as the convergence criterion is established.

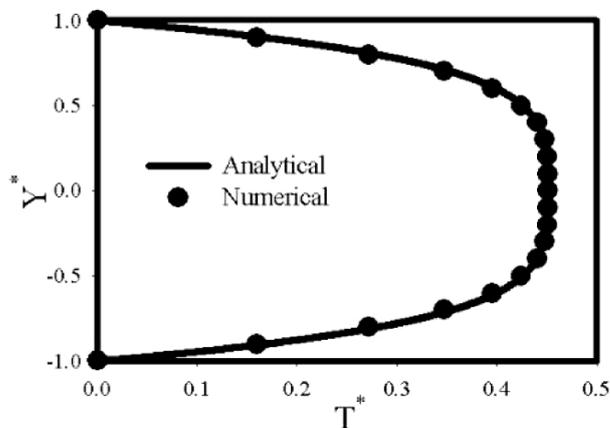
### 3- Results and Discussion

The inertial and non-isothermal flow of viscoelastic fluid has been simulated between two parallel plates by the simplified form of phan thien-tanner model and regardless of the temperature-dependency for fluid properties to validate

the results. The temperature distribution of current study has been compared with the analytical solution of Coelho et al. [8] in Fig. 2. According to this Fig., the result of the current study has a good agreement with the previous research.

In Fig. 3, the effect of Brinkman number changes on the distribution of local Nusselt numbers related to the lower and upper walls of the channel expanded part is examined for the inertial and non-isothermal flow of viscoelastic fluid. By forming the asymmetric recirculation regions, viscous dissipations and heat generation are increased and the maximum value of the local Nusselt number is located at the end of the first vortex in the vicinity of the channel lower wall.

Forming the asymmetric vortices and decreasing the viscosity (due to the increment of temperature) causes an increase in the velocity and high elongation of the fluid elements in the central district of the channel; thus, high heat is generated and stored in this region. By moving the fluid flow along the channel expanded part and finishing the recirculation regions, the heat stored inside the fluid structure is transferred from the central district to the walls of the channel. Therefore, the local Nusselt numbers are increased after the ends of vortices and are fixed in the thermally fully developed zone. Also, by raising the value of Brinkman number, more thermal energy is produced, so, in the hydrodynamic and thermally developing and fully developed zones, the local Nusselt numbers are increased.



**Fig. 2. Comparison of temperature distribution related to the numerical solution with the analytical solution**

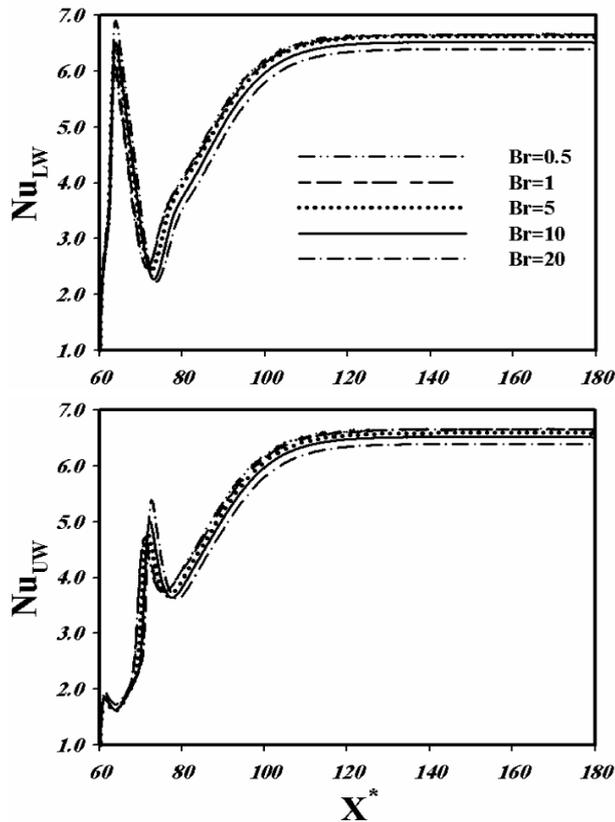


Fig. 3. The effects of Brinkman numbers on the distributions of local Nusselt numbers related to the lower and upper walls of the channel expanded part

#### 4- Conclusions

The results of this study show that for the hydrodynamic and thermally developing zone, the maximum values of the

local Nusselt numbers related to the lower and upper walls of the channel expanded part are located at the end of the first and second vortices respectively. Also for hydrodynamic and thermally developing and fully developed zones, the local Nusselt numbers are increased by incrementing the Brinkman number.

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