



## A Power-law Preconditioning Approach for Accelerating the Convergence Rate of Steady and Unsteady Incompressible Turbulent Flows

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**ABSTRACT:** In the present study, for the first time, the locally power-law preconditioning method for analyzing steady and unsteady incompressible turbulent flows around airfoils in high Reynolds numbers is utilized. In this method, the governing equations are modified by altering the time derivatives terms. The governing equations are discretized by the numerical method derived from the cell-centered Jameson's finite volume algorithm. In addition, for solving the unsteady flows, an implicit dual-time procedure and for simulating the turbulent flows, Baldwin and Lomax algebraic model have been employed. The computations are presented for steady and unsteady turbulent flows around NACA0012 and ONERA-A airfoils at various angles of attack and Reynolds number. Results presented in the paper focus on the velocity, pressure and eddy viscosity profiles, distribution of pressure coefficient, lift and drag coefficients and the effect of the power-law preconditioning method on the convergence rate. The numerical solution indicates an acceptable accuracy with the aid of the power-law preconditioning method in both steady and unsteady turbulent flows for high Reynolds numbers. Moreover, using the power-law preconditioning method improves the convergence speed significantly and reduces the iteration number of solution steps and central processing unit time simultaneously in both steady and unsteady flows.

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### 1- Introduction

An incompressible turbulent flow aerodynamics is one of the most profound and challenging areas in the field of fluid mechanics. A solution strategy is to scale down the acoustic wave speed in the same order of magnitude of convective wave speed artificially with pre-multiplying the time derivatives by a preconditioning matrix. This approach is known as preconditioning method. In recent years, there have been some activities in the field of development of preconditioning methods, in which the investigators try to increase the robustness and the accuracy of preconditioning methods such as the works of Malan et al. [1] and Esfahanian et al. [2].

The governing equations are modified by the locally power-law preconditioning method for the first time. Then, modified equations are solved with the use of Jameson's finite volume method. For time integration, explicit four-stage Runge-Kutta method is used and for solving unsteady flows, a dual-time implicit algorithm is applied. Furthermore, for simulation of turbulent flow, the Baldwin Lomax model is utilized. The numerical results are evaluated and compared with the existing results. The effect of preconditioning methods on the convergence rate is investigated. The results show that using the power-law preconditioning method improves the convergence speed significantly.

### 2- Governing Equations and Turbulent Model

Unsteady preconditioned governing equations of flow in non-dimensional and vector form are written as follows:

$$\Gamma \frac{\partial \bar{Q}}{\partial t} + \Pi \frac{\partial \bar{Q}}{\partial \tau} + \frac{\partial \bar{F}}{\partial x} + \frac{\partial \bar{E}}{\partial y} = 0 \quad (1)$$

where,  $\bar{Q} = (p \ u \ v)^T$ ,  $t$  and  $\tau$  present the real-time and the pseudo-time, respectively. The primitive variables are defined as:

$$\bar{F} = \begin{bmatrix} \rho u \\ p + \rho u^2 - \tau_{xx} \\ \rho uv - \tau_{yx} \end{bmatrix} \quad (2)$$

$$\bar{E} = \begin{bmatrix} \rho v \\ \rho uv - \tau_{xy} \\ p + \rho v^2 - \tau_{yy} \end{bmatrix}$$

where,  $\Pi^{-1}$  is the preconditioning matrix.  $\sigma$  is called preconditioning factor which is defined based on the locally power-law preconditioning method as follows:

$$\sigma = 0.5(1 - A_u)^m \quad (4)$$

The turbulent eddy viscosity is calculated in two different ways for the Baldwin Lomax model. There is a distinction between an inner layer and an outer layer. In the inner and outer layer, the viscosity are calculated with equations as follows [3]:

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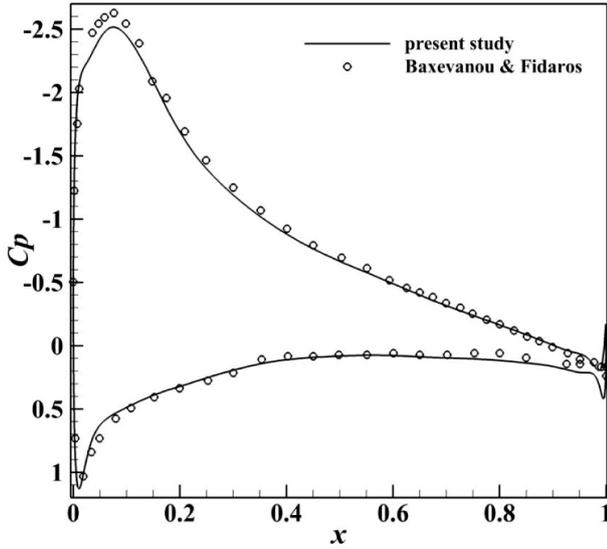


Fig. 1. Pressure coefficient distributions of steady turbulent flow over ONERA-A at AOA=8° and Re=5.25×10<sup>6</sup>

$$\mu_{t_{inner}} = \rho l^2 |\omega| \quad (5)$$

$$\mu_{t_{outer}} = \rho RC_{cp} F_{wake} F_{kelb}(y) \quad (6)$$

### 3- Numerical Discretization

In order to discretize the governing equations, Jameson's cell-centered finite volume method is employed. Initially, the preconditioned equation is integrated around the control volume  $\Omega$ , which is enclosed by the surface  $\partial\Omega$  as follows:

$$\Pi^{-1}\Gamma \frac{\partial}{\partial t} \int_{\Omega} \bar{Q} dA + \frac{\partial}{\partial \tau} \int_{\Omega} \bar{Q} dA \quad (7)$$

$$+\Pi^{-1} \int_{\partial\Omega} (\vec{F} dx - \vec{E} dy) = 0$$

Using Jamson's dissipative term, discretized form of

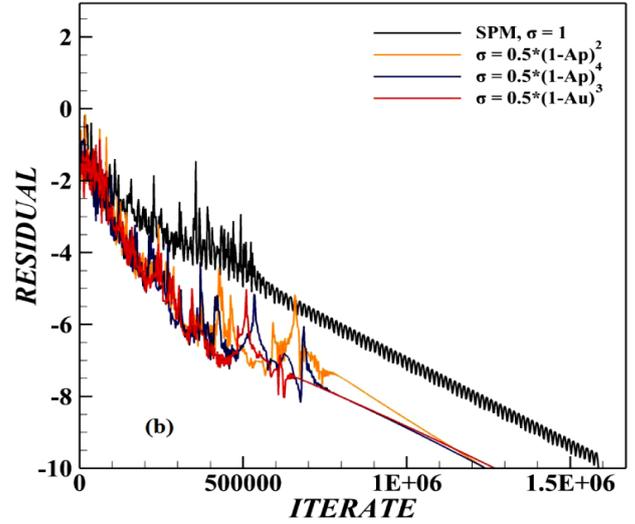


Fig. 3. Effect of the locally power-law PM on convergence rate of simulation for an steady flow over ONERA-A at AOA=8° and Re=5.25×10<sup>6</sup>

Eq. (12) becomes:

$$\Pi^{-1}\Gamma A_{i,j} \frac{\partial Q_{i,j,k}}{\partial t} + A_{i,j} \frac{\partial Q_{i,j,k}}{\partial \tau} = -G_{i,j,k} + D_{i,j,k} \quad (8)$$

where,  $A$  is the area of  $(i,j)^{th}$  cell,  $k=1,2,3$  is the index of primitive variable vector's components,  $D$  is the artificial dissipative term, and  $G$  is the numerical flux related to  $(i,j)^{th}$  cell.

### 4- Numerical Results

The main purpose of this paper is to investigate the convergence rate of power-law preconditioning methods. For this aim, the steady and unsteady incompressible turbulent flows over two airfoils, i.e. NACA0012 and ONERA-A are taken into account. To validate the proposed methods,

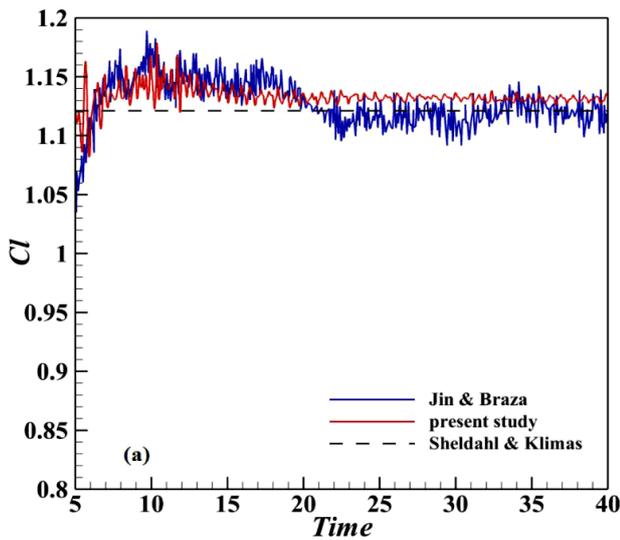


Fig. 2. Lift coefficient variation of unsteady turbulent flow over NACA0012 at AOA=12° and Re=1.0×10<sup>6</sup>

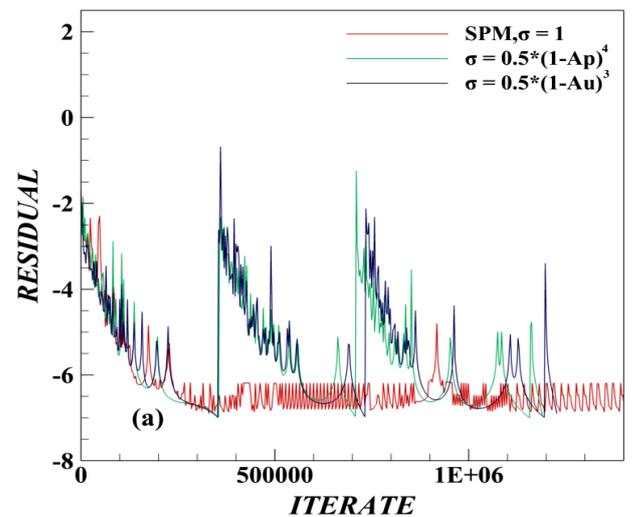


Fig. 4. Effect of the locally power-law PM on convergence rate of simulation for an unsteady flow over ONERA-A at AOA=5° and Re=1.0×10<sup>6</sup>

for steady flow, the pressure coefficient distributions over ONERA-A and for unsteady flow, the lift coefficient variation over NACA0012 are depicted in Figs. 1 and 2, respectively. The results are compared with the experimental works of Baxevanou and Fidaros [4], Jin and Braza [5], and Sheldahl and Klimas [6]. Figs. 1 and 2 show an acceptable level of conformity to works of aforementioned references. Figs. 3 and 4 show the effect of locally power-law preconditioning on the convergence rate for steady and unsteady flows. It is evident from these figures that the use of locally power-law PM has led to a significant reduction in the number of iterations.

## 5- Conclusions

In this study, numerical solution of steady and unsteady incompressible turbulent flows are considered using the locally power-law preconditioning method for the first time. The obtained results have an acceptable agreement with the numerical and experimental results of other researchers. They also indicate that the locally power-law preconditioning method improves the convergence rate to a large extent in both steady and unsteady flows.

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