



# Fractional Order Adaptive Fuzzy Terminal Sliding Mode Controller Design for a Knee Joint Orthosis

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**ABSTRACT:** Recently, rehabilitation robots have attracted much research interest. A novel fractional order fuzzy terminal sliding mode control with fractional order adaptive law is proposed for a knee joint orthosis in this paper. Knee joint orthosis robot is used for assistive and rehabilitation purposes. A model incorporating the human lower-limb and orthosis based on the Lagrange equations is utilized. At first, a fractional order terminal sliding mode control is proposed against the uncertainties and external disturbances. Then an adaptive fuzzy controller is designed to eliminate the unwanted chattering phenomenon in the control signal. Then a nonlinear disturbance observer is utilized with fractional order terminal sliding mode control to improve the accuracy and speed of tracking and to reduce the effect of the uncertainties in muscular torque modeling on the system control. The stability of the closed-loop system including the proposed controller with nonlinear disturbance observer is proved by using fractional order extension of Lyapunov theorem. To define the coefficients of the adaptive fuzzy fractional order terminal sliding mode control and the coefficients of the fuzzy membership functions, the particle swarm optimization algorithm is used. The performance of the proposed controller is compared with conventional sliding mode control, nonlinear disturbance observer based sliding mode control and proportional integral derivative control.

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## 1- Introduction

Related to the increase of the elderly and the increase in diseases such as stroke and spinal cord injuries the rehabilitation and assistive robots have drawn a large amount of interest [1]. Orthosis Exoskeleton Intelligently Communicating and Sensitive to Intention (EICOSI) is mainly considered to assist patients to strengthen their muscles in a sitting position [2].

Some advantages of this article are the stability of the closed-loop system including the proposed controller with Nonlinear Disturbance Observer (NDO) is proved by using fractional order extension of Lyapunov theorem, a new fractional-order sliding surface is proposed, only seven rules in the rule base of the fuzzy system are used to have the very low computational burden, adaptive rules for Full-Order Terminal Sliding-Mode Controller (FOTSMC) are designed, the Particle Swarm Optimization (PSO) algorithm is utilized to adjust the parameters of the Fractional order Adaptive Fuzzy Terminal Sliding Mode Control (FOAFTSMC) and the parameters of the fuzzy membership function.

## 2- System Modeling

The orthosis EICOSI robot is driven by both the actuator and human thigh muscles. The dynamic model can be written as [3]:

$$J\ddot{\theta} = -\tau_g \cos \theta - A \text{sign} \dot{\theta} - B \dot{\theta} - K(\theta - \theta_r) + \tau + \tau_h \quad (1)$$

By defining the state variable  $X = x_1, x_2^T = [\theta, \dot{\theta}]^T$  the dynamic model (1) can be rewritten as:

$$\dot{X} = F_1(X) + G_1(X)u + G_2(X)d, d = \tau_h, u = \tau \quad (2)$$

## 3- Nonlinear Disturbance Observer Design

The uncertainty in the estimation of muscular torque is considered as an external disturbance. Hence in this section, a NDO is utilized to estimate the muscular torques. Consider the human input torque  $\tau_h$  as a disturbance  $d$  and assume  $\hat{\tau}_h$  as the estimation of human input torque  $\tau_h$  and  $\hat{d} = \hat{\tau}_h$ . A NDO is designed as follows [4]:

$$\hat{d} = z + p(x) \quad (3)$$

$$\dot{z} = L(-F_1(x) - G_1(x)u - G_2(x)(z + p(x))) \quad (4)$$

$$L = \frac{\partial p(x)}{\partial x} = k_1, k_2 \quad (5)$$

## 4- Fractional Order Adaptive Fuzzy Terminal Sliding Mode Controller Design

The following fractional order sliding surface is proposed to design the FOTSMC to make more degree of freedom and more robustness.

$$\begin{cases} s(t) = c_1 e_1(t) + {}^c D_t^{1-\alpha} e_2(t) - w(t) \\ w(t) = c_2 v_1(t) + v(t) \end{cases} \quad (6)$$

$$v(t) = \begin{cases} a_0 + a_1 t + a_2 t^2 + a_3 t^3 T \\ 0 \end{cases} \quad (7)$$

In Sliding Mode Control (SMC) design  $u_f(t)$  (Eq. (8)) is composed of two parts; equivalent control ( $u_{eq}(t)$ ) and switching control ( $u_s(t)$ ).

$$u_f(t) = u_s(t) + u_{eq} = \rho \text{sign}(s(t)) + u_{eq} \quad (8)$$

In the design of ( $u_{eq}(t)$ ), it is assumed that the fractional derivative of Eq. (6) is equal to zero. As a result, ( $u_{eq}(t)$ ) is chosen as follows:

$$u_{eq}(t) = c_1 {}^c D_t^\alpha e_1(t) - {}^c D_t^\alpha w(t) \quad (9)$$

To decrease the chattering phenomenon and to increase the robustness against external disturbances and to overcome uncertainties in the system dynamics, the adaptive control, fuzzy logic and sliding mode control are combined together [5]. Assume that the control gain  $\rho \text{sign}(s(t))$  is replaced by a fuzzy gain ( $\rho$ ). The new control input could be rewritten as:

$$u_f(t) = u_{eq}(t) + \delta + \rho \quad (10)$$

The fuzzy system for  $\rho$  ought to be a Single-Input and Single-Output (SISO) system, with  $s(t)$  as the input and  $\rho$  as the output. Based on our knowledge of fuzzy systems,  $\rho$  can be written as follows:

$$\rho = \theta^T \Psi(s(t)) \quad (11)$$

$${}^c D_t^\alpha \theta = \gamma \Psi(s(t)) s(t) \quad (12)$$

**Theorem 1:** Consider the knee-joint robotic orthosis system (1), if the Eq. (11) is substituted in the control law which is shown in Eq. (10), then the system will be stable and the sliding surface will converge to zero.

**Proof.** Consider a Lyapunov function candidate as:

$${}^c D_t^\alpha \theta = \gamma \Psi(s(t)) s(t) \quad (13)$$

By taking  ${}^c D_t^\alpha$  of both side of Eq. (13) and by using the Theorems presented in reference [6] one can obtain:

$${}^c D_t^\alpha V(s(t)) = \frac{1}{2} {}^c D_t^\alpha s^2(t) + \frac{1}{2} {}^c D_t^\alpha \tilde{d}^2(t) + \frac{1}{2} {}^c D_t^\alpha \tilde{\theta}^2 \quad (14)$$

By substituting the adaptive rule in Eqs. (11) and (3) into Eq. (14) one can conclude that:

$${}^c D_t^\alpha V(s(t)) \leq 0 \quad (15)$$

Therefore, the closed-loop system with a proposed controller is globally asymptotically stable

### 5- 5. Simulation Results

The performance of the proposed controller is compared with three different methods (classical SMC, NDO Based SMC and PID) and these comparisons can be seen in Figs. 1 to 3.

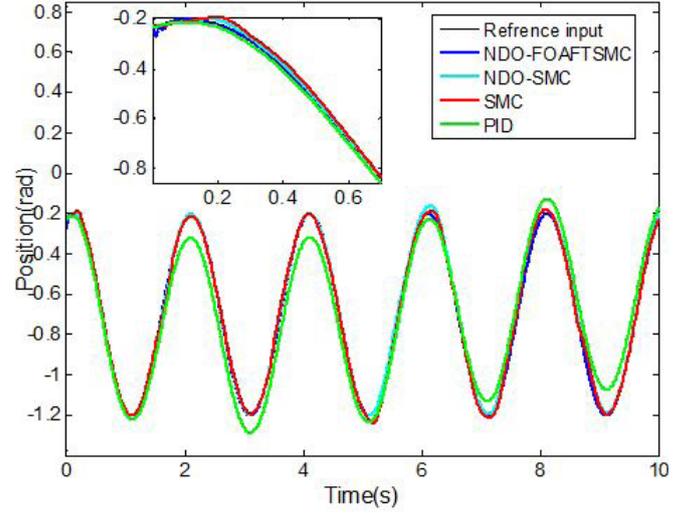


Figure 1. Comparison of tracking precision of NDO-FOAFTSMC, NDO-SMC, SMC, and PID.

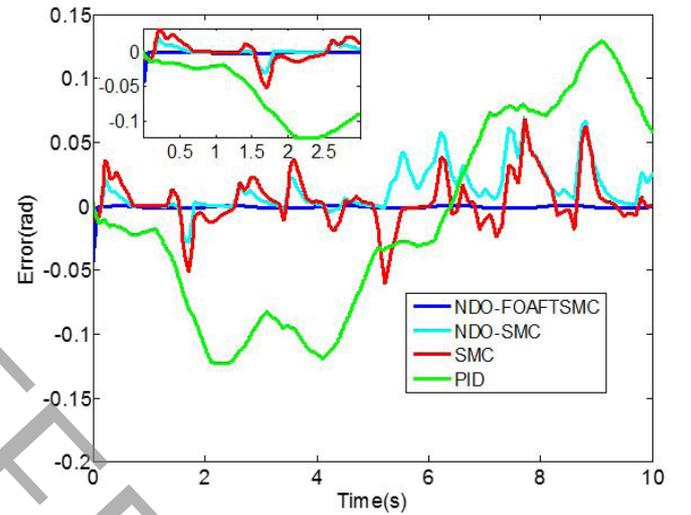


Figure 2. . Tracking error of NDO-FOAFTSMC, NDO-SMC, SMC, and PID.

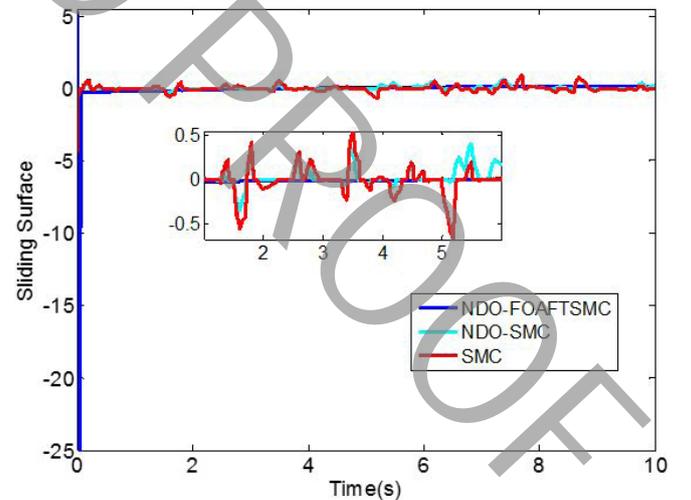


Figure 3. The sliding surface of NDO-FOAFTSMC, NDO-SMC, and SMC.

## 6- Conclusion

In this paper, human muscular torque was considered as an external disturbance, so the NDO based FOAFTSMC is designed to enhance the performance of the orthosis EICOSI. The stability of the proposed method was proved by using the fractional order extension of Lyapunov theorem. Simulation results confirmed that the proposed method improves the tracking precision, increase the speed of tracking and reduces the required time for eliminating the disturbance.

## References

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