



# Developing a Bidirectional Evolutionary Topology Algorithm for Continuum Structures with the Objective Functions of Stiffness and Fundamental Frequency with Geometrical Symmetry Constraint

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**ABSTRACT:** Topology optimization of structures, seeking the best distribution of mass in the design space to improve the performance and weight of a structure, is one of the most comprehensive issues raised in the field of structural optimization. In addition to the structure stiffness as the most common objective function, frequency optimization is of great importance in automotive and aerospace industries achieved by maximizing the fundamental frequency or the gap between two consecutive eigenfrequencies. The phenomenon of multiple frequencies, mesh dependency of topology responses, checkerboarding, geometric symmetry constraint, and occurrence of artificial localized vibration modes in low density regions are the most important challenges faced by the designer in stiffness and frequency optimization problems which influence the manufacturability of the design too. In this paper, a bidirectional evolutionary structural optimization method is applied for a frequency and stiffness problem separately via creating a software package including a Matlab code and Abaqus finite element solver linked to each other. Also, the effect of geometric symmetry constraint is considered on topologies arising from stiffness and frequency problems. So, the method is applied for modeling a 2 dimensional beam and its stiffness and frequency optimization and finally the optimization results of both objective functions will be compared with initial structure.

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## 1- Introduction

The main objective of structural optimization is to improve the functional and technological design of load bearing structures by considering objectives that are often times contradictory, like minimizing total mass or volume, minimizing stress, maximizing stiffness, maximizing fundamental frequency, etc. Topology optimization as the most comprehensive type of structural optimization is performed in the initial phases of the design process. The purpose is to determine the best material distribution in the design space, with respect to objective functions in order to improve the structural efficiency and reduce weight [1]. Various optimization methods such as homogenization [1], solid isotropic material with penalization parameter [2-6], evolutionary structural optimization [7, 8], and Level-set [9, 10] have been presented over the past decades. Evolutionary Structural Optimization (ESO) methods, is performed for discrete values of the design variable. It can be said that only two conditions are considered for a material used in the design space. In Bidirectional Evolutionary Structural Optimization (BESO) approach, first introduced by Yang et al. [11] in 1999, unlike the original methods that gradually eliminated unnecessary elements from the finite element model, the possibility of adding deleted elements was provided at the same time too.

In this paper, BESO method is modified and developed for

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frequency problems while solving it for a stiffness problem. The proposed Modified Bidirectional Evolutionary Structural Optimization (MBESO) is applied for stiffness and frequency optimization of a 2 dimensional beam via creating a software package including a Matlab code and Abaqus Finite Element (FE) solver linked to each other. Also, in this paper the effect of geometric symmetry constraint is considered on resulted topologies from stiffness and frequency problems and is considered as an effective factor in the convergence of the objective function for symmetric problems. Finally the optimization results of both objective functions will be compared with initial structure.

## 2- Topology Optimization Problem Statement

Topology optimization for stiffness objective function and a given volume of material is stated as:

$$\begin{aligned} C &= \frac{1}{2} f^T u \\ \text{Minimize:} & \\ \text{Subject to:} & \\ V^* - \sum_{i=1}^N V_i x_i &= 0 \\ x_i &= \{0 \text{ or } x_{\min}, 1\} \end{aligned} \quad (1)$$

where  $f$  and  $u$  are the applied load and displacement vectors and  $C$  is known as the mean compliance. If we assume that the design variable  $x_i$  continuously changes from 1 to  $x_{\min}$  (soft-kill approach) the sensitivity of the objective function

with respect to the change in the design variable is:

$$\alpha_i^e = \begin{cases} \frac{1}{2} u_i^T K_i^0 u_i & \text{when } x_i = 1 \\ 0 & \text{when } x_i = x_{\min} \end{cases} \quad (2)$$

The natural frequency  $\omega_j$  optimization problem can be stated as:

$$\begin{aligned} & \omega_j \\ \text{Maximize:} & \quad V^* - \sum_{i=1}^N V_i x_i = 0 \\ \text{Subject to:} & \quad x_i = \{x_{\min}, 1\} \end{aligned} \quad (3)$$

An alternative material interpolation scheme can be expressed as below to solve the artificial localized vibration modes in the low density regions:

$$\begin{aligned} \rho(x_i) &= x_i \rho_e^1 \\ E(x_i) &= \left[ \frac{1-x_{\min}}{1-x_{\min}^p} x_i^p - \frac{x_{\min}^p - x_{\min}}{1-x_{\min}^p} \right] E^1 \end{aligned} \quad (4)$$

From Eq. (4), the sensitivity of the objective function  $\omega_j$  can be expressed as:

$$\alpha_i = \frac{1}{p} \frac{d\omega_j}{dx_i} = \begin{cases} \frac{1}{2\omega_j} u_j^T \left( K_i^1 - \frac{\omega_j^2}{p} M_i^1 \right) u_j & x_i = 1 \\ -\frac{\omega_j^2}{p} u_j^T M_i^1 u_j & x_i = x_{\min} \end{cases} \quad (5)$$

### 3- Sensitivity Number Improvement

In addition to the sensitivity filter scheme [12] used in the BESO method in order to solve numerical problems, based on the computer experience, averaging the elemental sensitivity number in one or two successive steps in the optimization process, improves numerical instabilities:

$$\begin{aligned} \text{First Step:} & \quad \alpha_i = \frac{\alpha_i^k + \alpha_i^{k-1}}{2} \\ \text{Second Step:} & \quad \hat{\alpha}_i = \frac{\hat{\alpha}_i^k + \hat{\alpha}_i^{k-1}}{2} \end{aligned} \quad (6)$$

### 4- Numerical Example

#### 4- 1- Topology design of a 2 dimensional beam for frequency and stiffness

In this example, a simply supported 2 dimensional beam structure shown in Fig. 1 is considered for a prescribed

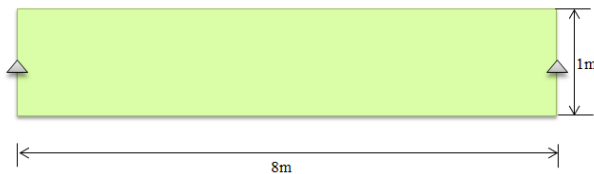


Fig. 1. Two dimensional design domain

volume fraction of  $V_f = 50\%$ . The rectangular design domain of  $8m \times 1m$  is divided into  $320 \times 40$  four node plane stress elements. Young's modulus  $E = 10$  MPa, Poisson's ratio  $\nu = 0.3$  and mass density  $\rho = 1$  kg/m<sup>3</sup>. A Newtonian force is also applied at the middle of lower edge for stiffness optimization. BESO parameters are selected as:  $ER = 2\%$ ,  $AR_{max} = 2\%$ ,  $x_{min} = 10^{-6}$ ,  $r_{min} = 0.075m$  and  $p=3$ .

#### 4- 2- Geometrical symmetry constraint

Due to the fact that elemental sensitivity numbers in stiffness optimization are of energy dimension, geometrically symmetric elements will have equal chance of elimination. In frequency problem sensitivity numbers of symmetric elements are not the same due to their angular dimension (Fig. 2).



Fig. 2. Non-symmetric first mode shape in 27<sup>th</sup>

In this paper the geometric symmetry constraint has been applied at the stage of removal / addition of elements.

### 5- Results and Discussion

Fig. 3 illustrates the final topologies of the structure for single-objective stiffness and frequency optimization.

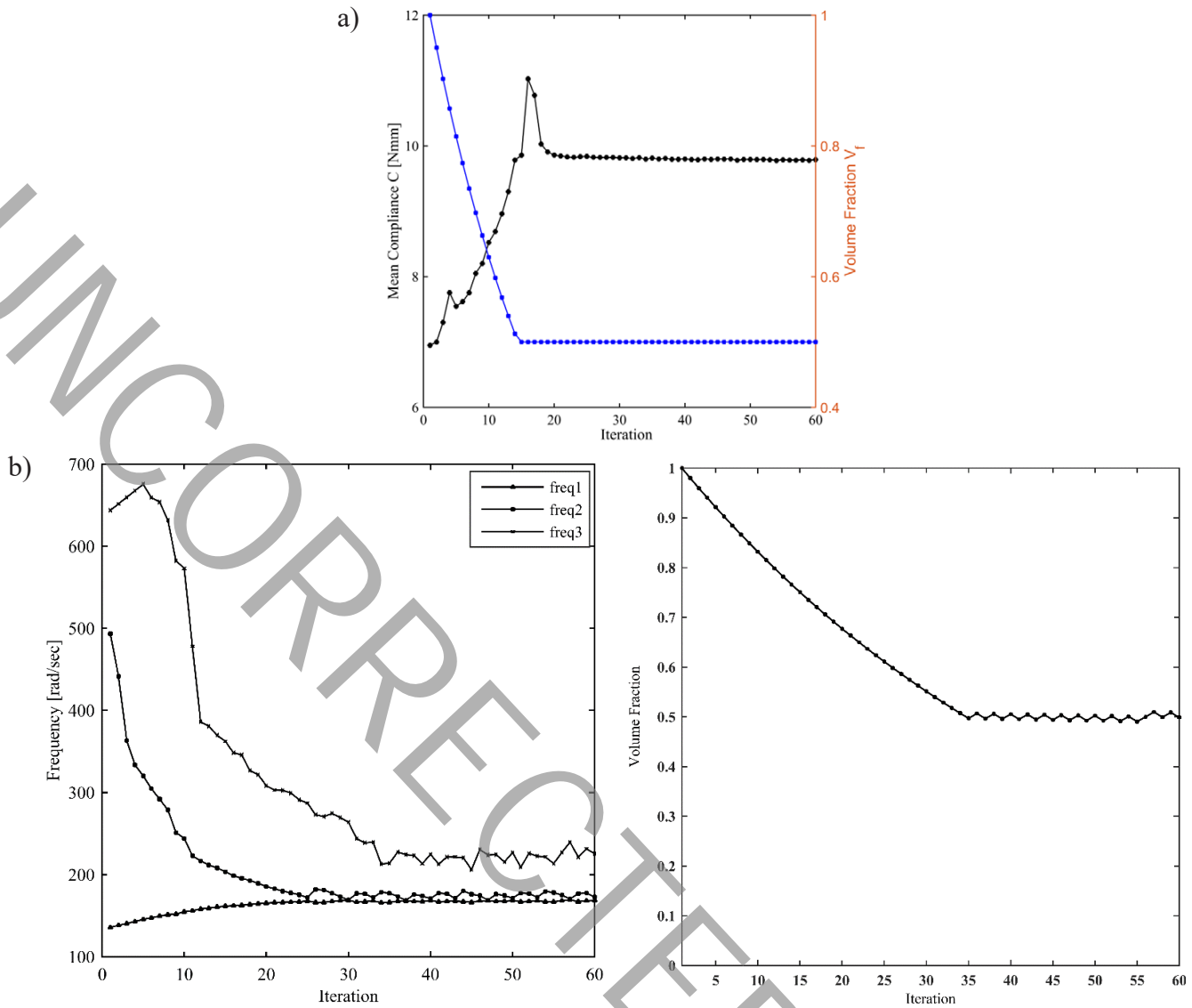


Fig. 3. Final topology of the 2 dimensional beam for maximum a) stiffness and b) fundamental frequency with 50% of volume fraction constraint.

Evolutionary history of the stiffness and frequency objective functions and their corresponding volume fraction is shown in Fig. 4.

### 6- Conclusions

In this paper, a modified BESO algorithm has been separately implemented for both stiffness and frequency objective function on a two-dimensional beam using a software package containing Matlab and Abacus linked to each other. As a result, while reducing the weight of the structure to half, the stiffness and natural frequency of the beam were maximized. It's seen that stiffness maximization leads to improvement of the frequency response which is a positive occurrence, but on the other hand the natural frequency maximization weakens the structure stiffness. This point must be considered when both bending stiffness and frequency are important.



**Fig. 4. Evolutionary history of the objective function and the volume fraction for a) stiffness b) frequency.**

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