Amirkabir Journal of Mechanical Engineering

Amirkabir J. Mech. Eng., 50(4) (2018) 1-3
DOI:

An Analytical Method for Damped Free Vibration Analysis of a Cracked Beam Considering the Coupled Multimode Equations

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ABSTRACT: The multimodal free vibration of a beam with a breathing crack excited by arbitrary initial conditions is investigated. Taking the initial conditions to be arbitrary makes more than one mode of the beam to be excited simultaneously. By considering the bending moment at the crack position, a multi-harmonic function describing the instantaneous opening and closing of the crack is extracted. Since the modal stiffnesses of the beam are dependent on the crack parameters, the extracted crack breathing function will appear in the equations of motion and makes them to be coupled. These equations are solved using the perturbation method. Then, the free response of the beam is extracted under three cases of initial conditions: excitation of the first mode, simultaneous excitation of the first and second modes, and simultaneous excitation of the first three modes. The results show that by exciting the first mode solely, the harmonic components of the response offer very limited information about the crack. However, by exciting the first several modes simultaneously, many other harmonic components appears at the frequency response curves which are more sensitive to the crack and contain more comprehensive information about the crack parameters.

Review History:

Received:
Revised:
Accepted:
Available Online:

Keywords:

Beam with a fatigue crack
Multimode free damped vibration
Perturbation method
Frequency analysis

1- Introduction

Vibration analysis of the beams to identify possible defects in them has been known as one of the main structural health monitoring methods. One of the most common defects in structures is the fatigue crack, which is generated mainly due to structural vibration, and if continued, will eventually lead to structural failure. The fatigue crack is usually open under tension and is closed under compression, so, in dynamic analysis of the cracked beam, the crack is modeled by the bilinear variation of the elastic force against the beam displacement [1], or by periodic variation of the elastic force with the time [2]. Because of the strong nonlinearity, in the literature, to perform analytical assessment of the vibration of the cracked beam, the dynamic model of the beam is usually simplified to a one degree of freedom system by considering only one mode of vibration of the beam. There are also very few references studying the multi-mode vibration of the cracked beam based on the finite element methods.

In this paper, the multimodal free vibration equations of the cracked beam are extracted as a set of coupled second order ordinary differential equations, and these equations are solved by the perturbation method [3]. The obtained analytical response makes it possible to assess the effects of crack parameters on the vibrational behavior of the beam directly and without any need to the numerical and or finite element based methods. The analytical free vibration responses of the cracked beam are obtained for the cases of single-mode and also multi-mode excitations; then by frequency decomposition of the responses, the harmonic components of the responses are extracted and the effects of the crack on them are studied.

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2- Multi-mode Response of the Beam with a Breathing Crack under an Arbitrary Initial Condition

A simply supported beam with a length of l and with a transverse crack located at l_1 is shown in Fig. 1.

Fig. 1. A simply supported cracked beam

When the beam is vibrating in one mode, it can be modeled as a single degree of freedom system governed by the following equation of motion [4]:

$$\bar{m}_n \ddot{u}_n(t) + \bar{c}_n \dot{u}_n(t) + \left[\bar{k}_n - \frac{\Delta \bar{k}_n}{2} \left(1 + \cos(\omega_n t)\right) \right] u_n(t) = 0 \quad (1)$$

where, $\bar{m}_n$, $\bar{c}_n$ and $\bar{k}_n$ are the equivalent mass, damping and elasticity of the intact beam. $\Delta\bar{k}_n$ is the difference between the elasticity of the intact beam and the cracked beam and ω_n is the n^{th} natural frequency of the cracked beam.

Eq. (1), although is the fundamental equation of many studies on the vibration of cracked beams, is only applicable in the case of single-mode vibration. For the multi-mode case the variation of the beam elasticity is no longer harmonic. In this case, because of the dependency of the beam elasticity on the status of the crack, in order to determine the variation of elasticity, the process of the crack opening and closing is to be modeled. If the crack opening is determined by the function $h(t)$, the equation of motion of the beam is determined as follows:

$$\bar{m}_n\ddot{u}_n(t)+\bar{c}_n\dot{u}_n(t)+\left[\bar{k}_n-\frac{\Delta\bar{k}_n}{2}\left(1+h(t)\right)\right]u_n(t)=0 \quad (2)$$

which may be nondimensionalized as follows:

$$\frac{d^2u_n}{d\tau^2}+2\varepsilon_n\mu_n\frac{du_n}{d\tau}+\left[1-\varepsilon_n\left(1+h(\tau)\right)\right]\alpha_n^2u_n(\tau)=0 \quad (3)$$

where, $\tau=\omega_1 t$, $\alpha_n=\dfrac{\omega_n}{\omega_1}$, $\varepsilon_n=\dfrac{\Delta\bar{k}_n}{2\bar{k}_n}$, $2\varepsilon_n\mu_n=\dfrac{\bar{c}_n}{\omega_1\bar{m}_n}$. The function $h(\tau)$ depends on the status of the crack, such that when the crack is fully open, $h(\tau)=1$, and when it is fully closed, $h(\tau)=-1$. The amount of crack opening is proportional to the magnitude of the bending moment at the crack location; so that:

$$h(\tau)\propto M=EI\frac{\partial^2 w}{\partial x^2} \quad (4)$$

where, M, E, I and w are the bending moment, modulus of elasticity, moment of inertia of the beam cross section and lateral displacement of the beam, respectively. By obtaining the first order approximation for the solution of the beam equation of motion and then using Eq. (4), $h(\tau)$ can be determined, which after normalizing, yields Eq. (5):

$$h(\tau)=\sum_{n=1}^{\infty}h_n\cos(\alpha_{nc}\tau) \quad (5)$$

where:

$$h_n=\frac{a_{n0}\phi_n''(l_1)}{\sum_{p=1}^{\infty}a_{p0}\phi_p''(l_1)} \quad (6)$$

where, ϕ_n is the n^{th} linear mode shape of the cracked beam and $a_{n0}=u_n(0)$.

By putting Eq. (5) into Eq. (3) and solving that by using the method of multiple scales [5], the analytical response of the cracked beam to an arbitrary initial displacement is obtained as follows:

$$\begin{aligned}w(x,t)&=\sum_{n=1}^{\infty}\phi_n(x)u_n\\&=\sum_{n=1}^{\infty}\phi_n(x)\Big\{a_n\cos(\omega_n t+\beta_n)\\&+\frac{1}{2}\varepsilon_n a_n\alpha_n^2\Bigg[\sum_{p=1}^{\infty}\frac{h_p}{\left(\omega_n^2-(\omega_n+\omega_{pc})^2\right)}\cos((\omega_n+\omega_{pc})t+\beta_n)\\&+\sum_{\substack{p=1\\p\neq 2n}}^{\infty}\frac{h_p}{\left(\omega_n^2-(\omega_n-\omega_{pc})^2\right)}\cos((\omega_n-\omega_{pc})t+\beta_n)\Bigg]\Big\}\end{aligned} \quad (7)$$

3- Case Study

Here, the multi-mode free response of the simply supported cracked beam is extracted for various cracks, and then, by frequency decomposition of the free response, the effects of crack parameters on the frequency response function of the cracked beam are studied. The mechanical properties of the beam are given in Table 1.

Table 1. The mechanical properties of the beam

Material	Length (cm)	Width (cm)	Thickness (cm)	Density (kg/m^3)	Young modulus (MPa)
AL 7075	56	2.54	0.64	2780	72400

In order to excite the fundamental mode of vibration of the beam, its initial displacement is considered to be proportional to the 1th mode shape, with the amplitude equal to 1 mm. The response is obtained for the beam with a crack located at the relative position of 0.25 and with the relative depths of 0 (intact beam), 0.3 and 0.6. In Figs. 2 and 3, the time responses of the midpoint of the beam as well as the frequency responses are shown respectively.

In the multi-mode case, the initial displacement of the beam is supposed to be a combination of three first linear mode shapes and with the ratios of 0.5, 0.3 and 0.2, respectively. The amplitude of the initial displacement is also taken equal to 0.8 mm. The crack parameters are taken as those in the single-mode case. The midpoint responses of the beam and also the frequency response functions for various cracks are shown in Figs. 4 and 5 respectively.

By comparing Figs. 2 and 4, one concludes that when only one mode is excited, the crack may sometimes have a slight effect on the system response, but in multi-mode excitation that is not the case, and the response changes significantly in presence of the crack. Also, considering Figs. 3 and 5, it is clear that in the multi-mode case, the crack causes several high-order crack sensitive harmonic components to be generated in the frequency response functions, and based on this fact, it will be possible to establish new crack detection methods based on the variations of the harmonic components of the cracked beam responses.

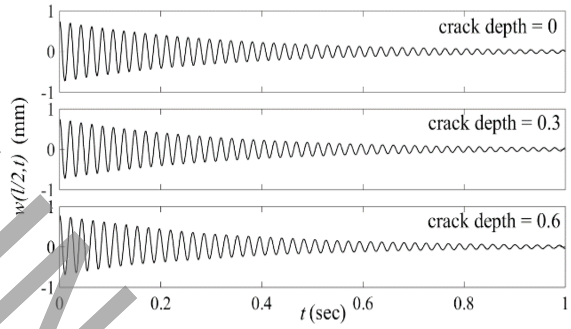

Fig. 2. Single-mode response of the cracked beam

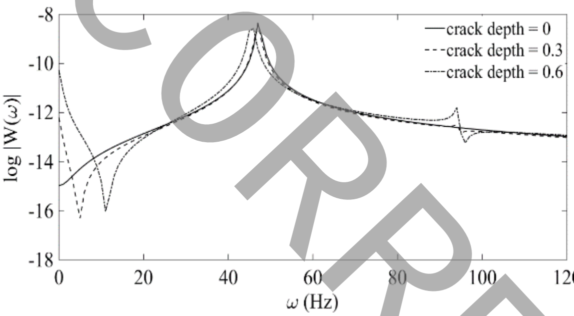

Fig. 3. Frequency response function of the cracked beam

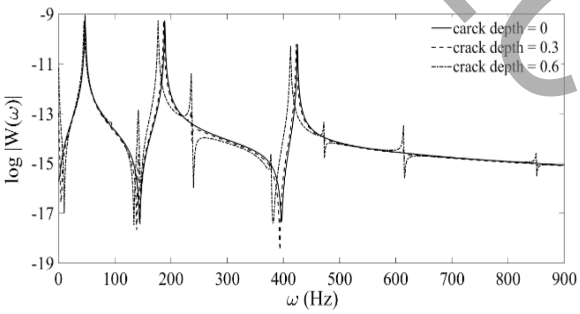

Fig. 4. Multi-mode response of the cracked beam

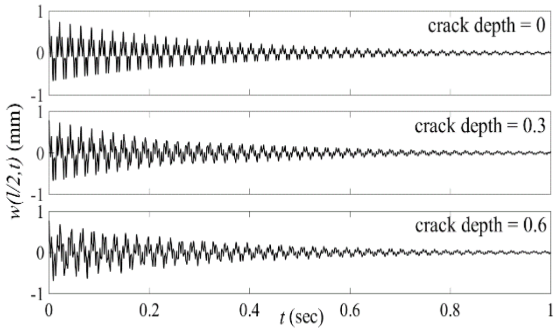

Fig. 5. Frequency response function of the cracked beam

4- Conclusions

In this paper, the multi-mode free damped vibration of a simply supported beam with a breathing crack is studied analytically. To this end, a multi-harmonic function representing the crack opening status as a function of the bending moment at the crack location is extracted. By using this function the multi-harmonic equations of motion of the cracked beam is obtained and solved analytically. The results show that when several modes are excited, the crack will produce many high-order harmonic components in the frequency response function, which may be used to establish new crack detection methods.

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