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Application of a Superelement in Static and Vibration Analysis of Piezoelectric Hollow Cylinders

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ABSTRACT: In this study, static and vibration analyses of cylindrical piezoelectric structures by means of superelements are targeted. In this regard, the cylindrical superelement is modified in order to be used in the analysis of hollow cylinders made of piezoelectric materials. At first, the cylindrical superelement, which was previously defined in the literature, is introduced. Next, the calculation of stiffness and mass matrices of piezoelectric structures in finite element analysis is briefly reviewed, and then, a piezoelectric cylindrical superelement is developed. In order to verify the accuracy of the defined element, two case studies are analyzed by means of the defined superelement, and the results are compared with the ones obtained by a commercial finite element software. In the end, the piezoelectric superelement is further modified to be used in static and vibration analyses of hollow cylinders which are made of functionally graded piezoelectric materials. Also, in this case, two classical problems are analyzed with the defined element. In both piezoelectric and functionally graded piezoelectric material cases, the results show appropriate compatibility with the ones obtained by the conventional elements.

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1. INTRODUCTION

Finite Element Method (FEM) is now widely used in academia and industry in order to analyze complicated engineering problems. In this method, a system in the first step should be discretized into the number of elements, which is called meshing. Based on the FEM, the accuracy of results is improved if the number of elements increased. However, it also increases the computation cost. Alternatively, researchers have tried to propose numerical methods which do not need meshing or at least need a lower number of elements [1, 2]. For example, in the automotive and aircraft industries, the sub-structuring method is frequently used. In this method, first, a structure with a complex shape has meshed, and then, by reducing the size of the assembled stiffness and mass matrices, a new element is defined which only includes nodes at boundaries [3-5]. As a result, a large element with a substantially reduced number of nodes and Degrees-Of-Freedom (DOFs) is created. Although this method reduces the computation time substantially, it still needs meshing.

The superelement method is another approach with which a structure can be analyzed efficiently with a smaller number of elements. In this method, an element with a known geometry is defined and customized based on the demanded analysis. For example, Ahmadian et al. [6] introduced a cylindrical superelement which can be used in the structural analysis of cylindrical laminates. Later, Taghvaeipour et al. [7] modified the element formulation to be used in the analysis of functionally graded thick vessels.

This study aims at the definition of a cylindrical superelement which can be incorporated in static and vibration analysis of piezoelectric sensors/actuators. In this regard, the superelement which was first defined in Ref. [6] is briefly introduced, and then, the stiffness and mass matrices of a cylindrical piezoelectric superelement are derived. Finally, by using the defined element, some examples are solved and the results are verified by a commercial FE package.

2. THE CYLINDRICAL SUPERELEMENT FOR PIEZOELECTRIC MATERIALS

The geometry of the cylindrical superelement is depicted in Fig. 1. As it is shown, this element with the inner radius of

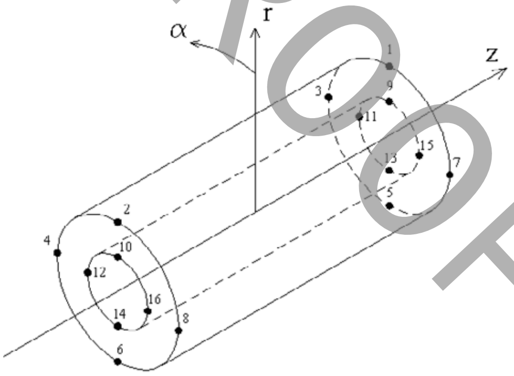

Fig. 1: The Cylindrical superelement

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r_1, the outer radius of r_2 and the length of $2L$ has 16 nodes. For the sake of simplicity, and based on the cylindrical geometry of element new local coordinates are defined as follows,

$$\xi = \frac{z}{L} \qquad \eta = \frac{2\times r - b}{a} \qquad \gamma = \frac{\alpha}{\pi} - 1 \tag{1}$$

in which:

$$a = r_2 - r_1 \qquad b = r_2 + r_1 \tag{2}$$

The DOFs are interpolated inside the element by means of 16 shape functions which are formed by trigonometric functions, along the radial direction, and polynomial functions along the radial and longitudinal directions [6]. For an electromechanical analysis, each node possesses four DOFs, and hence, the nodal vector of the i'th node is defined as:

$$\mathbf{q}^i = \begin{bmatrix} u_{ir} & u_{i\alpha} & u_{iz} & \phi_i \end{bmatrix}^T \tag{3}$$

In piezoelectric materials, the governing equations are summarized as follows:

$$\{T\} = [c^E]\{S\} - [e]^T\{E\} \qquad \{D\} = [e]\{S\} + [\varepsilon^S]\{E\} \tag{4}$$

where $\{T\}$ is denoting the stress vector, $\{S\}$ is the strain vector, $\{E\}$ is the electrical field, $\{D\}$ is the electrical displacement, $[\varepsilon^S]$ is the matrix of elastic coefficients while the electrical field is constant, and $[e]$ is the matrix of piezoelectric coupling coefficients. By resorting to Hamilton's principle and approximation solution, the following matrix equations are obtained within each element,

$$[M]\{\ddot{u}_i\} + [K_{uu}]\{u_i\} + [K_{u\Phi}]\{\Phi_i\} = \{f_i\} \tag{5}$$

$$[K_{\Phi u}]\{u_i\} + [K_{\Phi\Phi}]\{\Phi_i\} = \{g_i\} \tag{6}$$

The corresponding formulations for the stiffness and mass matrices and the derivations are presented in Ref. [8].

3. THE CASE STUDIES

The First case study is a clamped-free hollow cylinder with the ratios of $L/r_i = 8$, $h/r_i = 0.5$ and the thickness of $h = 0.01$ m. The cylinder is made of PZT-4 which is polarized along its length. The elongation of the foregoing structure under the following boundary conditions are obtained by the superelements, and brick elements in a FE commercial software. The results are compared in Table 1.

Table 1: The elongations obtained by superelements (SE) and brick elements

	Elongation (m)	Difference%
Brick Elements	2.83e-8	-
SE 1 Element	2.2883e-8	19.1
SE 3 Elements	2.6428e-8	6.6
SE 5 Elements	2.7295e-8	3.55
SE 10 Elements	2.7929e-8	1.31

Table 2: The first two bending natural frequencies obtained by superelements and brick elements (short-circuit)

	1st Bending (Hz)	Diff%	1st Torsion (Hz)	Diff.%
Brick Elements	682.27	-	2414.2	-
10 SEs	710.86	4.19	2523.9	4.55
15 SEs	699.73	2.56	2522.1	4.47
20 SEs	696.3	2.06	2521.5	4.44

$$\Phi = 100\text{V} \rightarrow z = 0 \qquad \Phi = 0 \rightarrow z = L \tag{7}$$

In the second case study, a hollow cylinder with the ratios of $L/r_i = 20$, $h/r_i = 2$ and the thickness of $h = 0.02$ m is considered. Likewise, the cylinder is clamped-free and made of PZT-4. In the case of short-circuit boundary condition, the first two bending natural frequencies obtained by the superelement and the brick elements are compared in Table 2.

4. CONCLUSIONS

In this study, a cylindrical superelement was introduced which can be used in the structural analysis of hollow cylinder sensors/actuators. The element is based on the geometry and shape functions which were previously defined in the literature, and here, it is developed to be used in the case of piezoelectric hollow cylinders. In the end, static and vibration analysis were conducted on case studies by means of superelements. The results show proper accuracy compared with the ones obtained by brick elements in a commercial FE software.

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