



## ***Investigating Transverse Vibrations of Free-Free Beam on the Frictional Substrate and Validation with Test Results***

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### **ABSTRACT**

In the present paper, the transverse vibrations of the Euler–Bernoulli beam along with the strong dry friction at the boundary conditions have been investigated. With special attention to the force of friction on the boundary conditions which plays a role as one non-linear factor and brings out nonlinear response, natural frequencies also change despite the linear systems and these natural frequencies become a function of the force of friction and the force of friction in substrates is also a function of other parameters. The calculation of the beam frequencies as well as proper modeling of boundary conditions have been interesting for researchers and scientists. In the present study, the force of friction at substrates is investigated through the friction force modeling by using the nonlinear elastic-plastic Valanis model and also by the Classical Model of Coulomb friction. The assumed modes method for solving is used and the resulting equations by using the Fourth Runge-Kutta numerical method are solved. These two models are compared and the results of modal testing are used for the validation. Finally, according to Valanis parameters, it is shown that this model has a positive relation with the findings. The hardening effect of the frequency response-curves is also observed.

### **KEYWORDS**

Nonlinear Vibrations of Beam, Valanis Model, Coulomb Model, Modal Testing.

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### 1- INTRODUCTION

The vibrational behavior of structures is strongly affected by the boundary conditions, joints and interfaces. Usually the non-linear factors caused by the frictional contacts in the boundary conditions changes the dynamic response of the structure and introduce unknown physics to the problem, which results in some inaccuracies in the theoretical analysis predictions. The friction force as an important non-linear factor in contact points is a function of various factors, which depend on the area of contact. For low amplitude vibration, static friction dominates, and frictional effects are in the stick regime. Theoretically speaking, the accurate modeling of the friction phenomena leads to more accurate theoretical predictions of the structures vibrational behaviors. Therefore, the more a model can simulate the friction non-linear effects, the more efficient that model is [1-3].

In this paper, the non-linear factors related to the friction forces are excited by different amplitudes and the friction is kept in the stick regime by controlling the excitations amplitudes. Then a suitable contact model is used to model the non-linear behavior of the contact to determine the friction forces in different vibration levels. It is found by using the Valanis elastic-plastic model [4] and estimating its parameters using the experimental results, an appropriate frequency response is obtained this model is well validated by the results achieved through experiment. The friction model's parameters are extracted through experimental studies under different force levels conditions, as predicted.

### 2- MATHEMATICAL- MODELING

Fig. 1 shows a slender beam mounted on frictional contacts at both ends. Based on the following assumptions 1) the beam is modeled based on the Euler-Bernoulli beam theory, 2) the displacements are small compared with beam dimensions, 3) the cross section area, moment of inertia, density and modulus of elasticity are uniform through the beam's length, 4) the axial inertia is negligible (since the excitement's frequency domain is far enough from the beam's axial frequency domain). The non-linear governing equation of the beam's lateral vibrations is given below

$$\begin{aligned}
 m\ddot{w} + EIw^{(4)} = & -M\ddot{w}(0,t)\delta(0) \\
 & - f(t)\delta(0) \\
 & + rN_2\delta'\left(-\frac{L}{2}\right) \\
 & + rN_2\delta'\left(\frac{L}{2}\right) \rightarrow -L/2 < x \\
 & < L/2
 \end{aligned} \tag{1}$$

The friction force in the above equation can be substituted by the following non-linear Valanis model

$$\dot{N}(t) = \frac{e_0 u \left[ 1 + \frac{\lambda}{e_0} \operatorname{sgn}(\dot{u})(e_t u - N(t)) \right]}{1 + \kappa \frac{\lambda}{e_0} \operatorname{sgn}(\dot{u})(e_t u - N(t))} \tag{2}$$

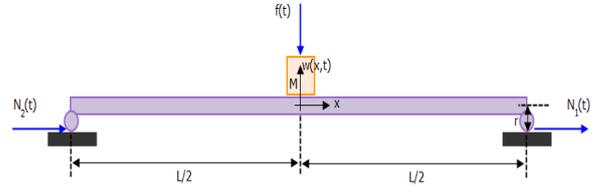


Fig. 1. Schematic of beam with two frictional contacts

The obtained equations are solved by the fourth-order Runge-Kutta method. Considering the dominant effect of the lower frequency modes on the beam's vibrations, a few modes are used to construct the numerical model. Fig. 2 shows the typically results obtained using three modes easily observed that it also the second and upper modes have less than one percent effect on the results.

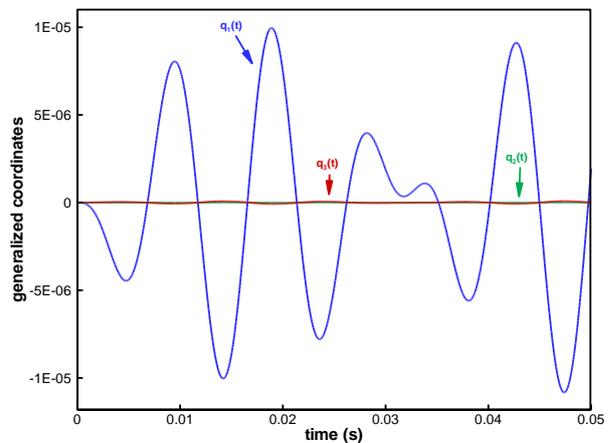


Fig. 2. Generalized coordinates ( $F / F_0 = 1.5$ ).

A steel beam with a length of 40cm, width of 4cm and thickness of 1cm is used in which two ends lie on a frictional contact interface. Two cylindrical steel pieces with a radius of 1.5cm as wide as the beam are welded to both ends. The frictional contact interface is a thin layer of abrasive. The experimental setup is shown in Fig. 3.



Fig. 3. Test set-up

### 3- RESULTS AND DISCUSSION

The plots of measured acceleration amplitude versus the excitation frequency for four excitation force levels are shown in Figs. 4. The dissymmetry of the plots increases as the force level increases due to the growing of nonlinearities in the beam's supports. As seen in Fig. 4, the peak points of the frequency response tend to the right as the force level increases. This shows the hardening effects due to the existence of the friction in the boundary conditions. The maximum response amplitude increases as the force level increases.

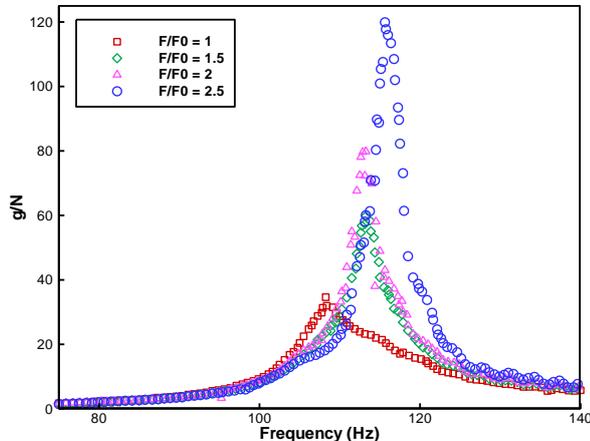


Fig. 4. Measured frequency responses for four force amplitudes

### 4- CONCLUSION

The effects of the friction force at the supports on the frequency response of a beam were fully investigated. The beam was excited by different force levels and the variations of frequency and amplitude of vibration due to the non-linear effect of the friction forces were considered. It is observed that the abrasive contact interface may cause unstable vibrations. For there more, the performed studies show that the numerical model based on the Euler-Bernoulli beam theory in combination with the Valanis non-linear elastic-plastic friction model can be of good use for appropriately predicting the obtained experimental data.

### 5- REFERENCES

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