Amirkabir Journal of Mechanical Engineering

Amirkabir J. Mech. Eng., 53(1) (2020) 1-3
DOI:

# Modeling of Hyperelastic Incompressible Behavior of Functionally Graded Material under Bending Load

G. H. Rahimi*, M. M. Memarianm, Y. Anani, S. Hosseini Chaleshtori

Department of Mechanical Engineering, Tarbiat Modares University, Tehran, Iran



**ABSTRACT:** In this paper, the behavior of inhomogeneous functionally graded rubber with large deformations and under bending loading is modeled by assuming an incompressible hyper-elastic material. For modeling the nonlinear behavior of the material, hyperelastic theory and strain energy functions were used. The strain energy is a function of the left Cauchy-Green deformation tensor invariants. The constants of strain energy are considered as power and in direction of curvature radius. Also, the generalized Mooney-Rivlin function was used for modeling the nonlinear behavior. Supposing the power constants of strain energy is convenient for description of material behavior. Also the results of the analytical solution are compared to those of Finite Element method and there is acceptable accuracy.

**Review History:**

Received:
Revised:
Accepted:
Available Online:

**Keywords:**

Hyperelastic material
Incompressible
Bending behavior of rectangular cross-section
Inhomogeneous functionally graded material Il

## 1- Introduction

Different groups of materials, such as foams, elastomers, biological tissues, and polymers are the nonlinear hyperelastic materials. Reversibility is the most important physical property of hyperelastic materials. Natural rubber sometimes stretches up to eight times its original length and then comes back to its original state. Typical Natural rubbers are bitumen, turtle outer cover, animal's antler, and gum trees and artificial rubber such as polybutadiene, styrene-butadiene, nitrile, butyl, etc. Different types of materials, such as rubber, are reversible in large deformation. The maximum value of the stretch is usually between 5-10 (the ratio of the current length to the initial length) and the stress-stretch curve is non-linear. So the material does not follow Hooke's law. To modeling on the behavior of these materials, the material is considered as a continuous environment, and a strain energy density function is obtained, which is usually in terms of the deformation invariant [1].

## 2- Methodology

The geometry of the section before and after deformation shown in Fig. 1. The displacement fields are:

$$r = f(X), \quad \theta = \frac{Y}{\rho}, \quad z = Z. \tag{1}$$

which $r, \theta, z$ and X, Y, Z are Cartesian and cylindrical coordinates, respectively.

*Corresponding author's email: rahimi_gh@modares.ac.ir

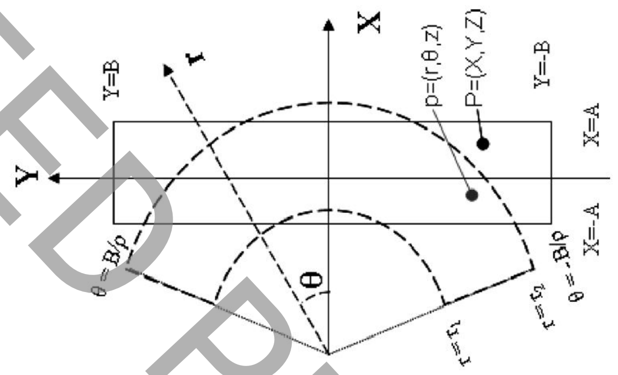


**Fig. 1. The rectangular section before deformation and the circular section after deformation [2]**

Using the Rivlin method [3], the constitutive method for incompressible and isotropic hyperelastic materials is expressed as follows:

$$\boldsymbol{T} = -p\boldsymbol{I} + 2\frac{\partial W}{\partial I_1}\boldsymbol{B} - 2\frac{\partial W}{\partial I_2}\boldsymbol{B}^{-1} \tag{2}$$

where p is hydrostatic pressure depends on the incompressibility constraint, T is the Cauchy stress and I is the unit matrix. $W = W(I_1, I_2)$ , is the strain potential energy function, which is based on $(I_1 - 3)$ and $(I_2 - 3)$. Mooney-Rivlin strain energy is the classical stress energy density for homogeneous incompressible rubber:

$$W^{MR} = C_1(I_1 - 3) + C_2(I_2 - 3), \tag{3}$$

which $C_1, C_2$ are the constants of the Mooney-Rivlin energy function. For functionally graded materials, the Mooney-Rivlin energy function is $W = \mu(r)(I_1 - 3) + \mu'(r)(I_2 - 3)$, which $\mu$ indicates the shear modulus of the material in the deformed configuration. For the functionally graded inhomogeneous material, $\mu(r) = \mu_{10}\left(\frac{r}{r_2}\right)^n, \mu'(r) = \mu_{01}\left(\frac{r}{r_2}\right)^m$ which n and m are inhomogeneous coefficients of material. Therefore, the generalized Mooney-Rivlin strain energy function for inhomogeneous functionally graded materials is:

$$W = \mu_{10}\left(\frac{r}{r_2}\right)^n (I_1 - 3) + \mu_{01}\left(\frac{r}{r_2}\right)^m (I_2 - 3) \qquad (4)$$

## 3- Solving method

The equilibrium equations along the radius and in the absence of volumetric forces are simplified as follows [2]:

$$\frac{\partial T_{rr}}{\partial r} + \frac{1}{r}(T_{rr} - T_{\theta\theta}) = 0, \quad \frac{\partial T_{\theta\theta}}{\partial \theta} = 0, \quad \frac{\partial T_{zz}}{\partial z} = 0 \qquad (5)$$

To calculate the main stresses, Eq. (5) is integrated and after simplifying, the Cauchy stress relations for inhomogeneous functionally graded materials are obtained:

$$T_{rr}(r) = \frac{2\mu_{10}\rho^2}{(n-2)r_2^n}\left(r_1^{n-2} - r^{n-2}\right) + \frac{2\mu_{01}}{(m+2)r^2 r_2^m}\left(r^{m+2} - r_1^{m+2}\right) + \frac{2\mu_{10}}{(n+2)\rho^2 r_2^n}\left(r^{n+2} - r_1^{n+2}\right) + \frac{2\mu_{01}\rho^2}{(m-2)r_2^m}\left(r_1^{m-2} - r^{m-2}\right). \qquad (6)$$

$$T_{\theta\theta}(r) = -\frac{2\mu_{10}\rho^2}{r_2^n}\left(r^{n-2} + \frac{r^{n-2}}{n-2} - \frac{r_1^{n-2}}{n-2}\right) - \frac{2\mu_{01}}{\rho^2 r_2^m}\left(-r^{m+2} + \frac{r_1^{m+2}}{m+2} - \frac{r^{m+2}}{m+2}\right) - \frac{2\mu_{10}}{\rho^2 r_2^n}\left(\frac{r_1^{n+2}}{n+2} - \frac{r^{n+2}}{n+2} - r^{n+2}\right) - \frac{2\mu_{01}\rho^2}{r_2^m}\left(\frac{r^{m-2}}{m-2} - \frac{r_1^{m-2}}{m-2} + r^{m-2}\right) \qquad (7)$$

$$T_{zz}(r) = -\frac{2\mu_{10}\rho^2}{r_2^n}\left(r^{n-2} + \frac{r^{n-2}}{n-2} - \frac{r_1^{n-2}}{n-2}\right) - \frac{2\mu_{01}}{\rho^2 r_2^m}\left(-r^{m+2} + \frac{r_1^{m+2}}{m+2} - \frac{r^{m+2}}{m+2}\right) - \frac{2\mu_{01}}{r_2^m}\left(\frac{\rho^2 r_1^{m-2}}{m-2} - \frac{\rho^2 r^{m-2}}{m-2} + r^m\right) - \frac{2\mu_{10}}{r_2^n}\left(\frac{r^{n+2}}{(n+2)\rho^2} - \frac{r_1^{n+2}}{(n+2)\rho^2} - r^n\right). \qquad (8)$$

## 4- Results and Discussion

The results of the exact solution are compared with those of modeling in ABAQUS software. CPE8RH element has been used in ABAQUS modeling. The comparison between exact solution and numerical solution for stresses along radius of the section are shown in Figs. 2 to 4 for radial, circumferential and axial stresses respectively.

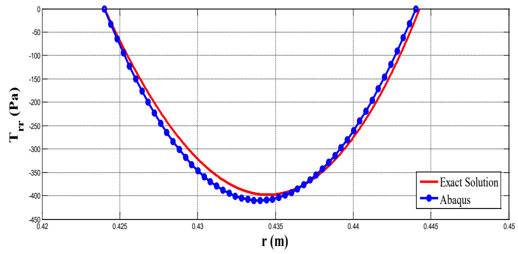


**Fig. 2. Comparison of theoretical and numerical solution results for radial Cauchy stress**

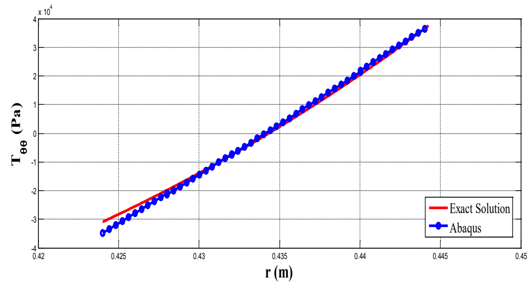


**Fig. 3. Comparison of theoretical and numerical solution results for circumferential Cauchy stress**

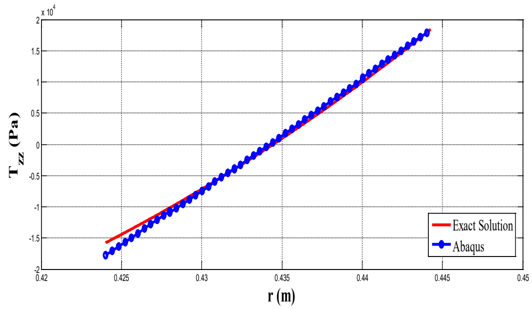


**Fig. 4. Comparison of theoretical and numerical solution results for axial Cauchy stress**

As a result, new equations are obtained for describing the behavior of the incompressible hyperelastic rectangular cross-section of inhomogeneous isotropic under bending using analytical solution. By comparing the exact solution method used in this paper to the numerical model, it is concluded that these relations have good accuracy. Therefore, it can be concluded that simplifying assumptions such as zero shear stress, are good and correct assumptions. The advantage of this article is that it integrates into equilibrium equations as direct and there are no errors due to numerical methods.

## 5- Conclusions

In this research, the modeling of the hyperelastic behavior of inhomogeneous functionally graded rubber under bending loading and extracting the Cauchy stress relations of the cross-sectional by this loading has been made. For modeling, the generalized Mooney-Rivlin energy function has been used and the properties were changing into radius. Also the property was inhomogeneous. Finally, the analytical results are compared to those of the numerical results and it is shown that the functions are described the behavior of the hyperelastic materials under pure bending.

## 6- References

[1] Y. Anani, G.H. Rahimi, Modeling of hyperelastic behavior of functionally graded rubber under mechanical and thermal load, Modares Mechanical Engineering, Vol. 15,(No. 11,) (2015) pp. 359-366.

[2] L. M. Kanner, C.O. Horgan, Plane strain bending of strain-stiffening rubber-like rectangular beams, International Journal of Solids and Structures, 45(6) (2008) 1713-1729.

[3] Y. B. Fu, R.W. Ogden, Nonlinear Elasticity: Theory and Applications, Cambridge University Press, London, (2001).