



Numerical Solution of Viscoplastically Lubricated Multi-layer Core-Annular Flow Using the Spectral Element Method

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ABSTRACT: The aim of this research is to simulate a multi-layer flow of the core-annular type in a two-dimensional channel, in which a Newtonian fluid in the core is surrounded by a viscoplastic fluid of the regularized Bingham type. This simulation is based on the volume of fluid method. Flow and concentration equations are discretized spatially by the spectral element method. The velocity correction scheme, as a high order algorithm, is developed for splitting the velocity and pressure variables. Considering a developed flow leads to a nonlinear equation in the plastic region of the flow, which is numerically solved and is called semi-analytic solution and along with the previously published works, is used to validate the spectral element results. The effect of the main parameters of the flow, i.e. Bingham number, viscosity ratio and core thickness on the pressure drop and un-yielded region thickness is evaluated. The results show that the Bingham number is the most effective parameter on the pressure drop and un-yielded region thickness. Also the profiles of secondary variables, including apparent viscosity and shear stress, across the channel section are presented and show that in the interface of the fluids, there is a difference between numerical and semi-analytic solutions.

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1- Introduction

The core-annular arrangement of two immiscible fluids has applications in multi-layer extrusion, film coating, and heavy oil transportation. The stability of flow regime may be the most challenging problem in this field [1]. A proposed solution to overcome this problem is employing a viscoplastic fluid as the lubricant [2].

A viscoplastic or yield stress fluid is a kind of fluid which behaves like a solid until the loading stress reaches a threshold value, i.e. yield stress, τY , afterward it starts to show viscous behavior like a fluid. When the interfacial shear stress is smaller than the yield stress, an un-yielded solid-like layer will be formed in the boundary of two fluids and leads to suppression of possible instabilities.

In the present work, a high order splitting scheme, which is based on spectral element discretization, is developed to model the core-annular multi-layer flow with viscoplastic lubrication in a two-dimensional channel. Moreover, a semi-analytic solution for the developed flow region is developed and solved using Wolfram language and is used for validating the numerical solution. The effect of hydrodynamic parameters, i.e. inlet core thickness and viscosity ratio, and rheological parameters like Bingham number (dimensionless yield stress) on the un-yielded region thickness is evaluated and discussed.

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2- Methodology

2-1- Governing equations

The sketch of the present work may be seen in Fig. 1. The channel initially is filled with a viscoplastic fluid at the rest. At $t = 0$, a Newtonian fluid enters the channel through the core region, of thickness $2Y_i$, and simultaneously, the lubricant fluid is driven due to the applied pressure gradient through the outer layers (annulus). Both fluids have the same velocity at the inlet.

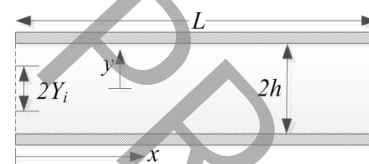


Fig. 1. Sketch of the problem

The volume of fluid method is used to model the two phase flow. In this method, the same momentum and continuity equations are solved for the whole domain and a concentration equation is employed to track the presence of each fluid in the whole region. Dimensionless governing equations are as:

$$\begin{aligned}
 u_{j,j} &= 0, \\
 \left[\frac{\partial}{\partial t} + u_j \frac{\partial}{\partial x_j} \right] u_i &= \frac{1}{Re} \tau_{ij} - P_i, \\
 \left[\frac{\partial}{\partial t} + u_j \frac{\partial}{\partial x_j} + \frac{1}{Pe} \frac{\partial^2}{\partial x_j \partial x_j} \right] C &= 0,
 \end{aligned} \tag{1}$$

In which u , P and C are velocity, pressure, and concentration, respectively and τ_{ij} is the deviatoric stress tensor that can be calculated by harmonic interpolation through:

$$\tau_{ij} = \left(\frac{C}{\tau_{ij}^{[1]}} + \frac{1-C}{\tau_{ij}^{[2]}} \right)^{-1} \quad (2)$$

The superscripts 1 and 2 are for core and annular fluids, respectively. The constitutive equation for the two fluids is as:

$$\begin{aligned} \tau_{ij}^{[1]} &= r_\mu \dot{\gamma}_{ij}, \\ \tau_{ij}^{[2]} &= \left(1 + \frac{Bn}{\sqrt{\dot{\gamma}^2 + \varepsilon^2}} \right) \dot{\gamma}_{ij} = \mu(\dot{\gamma}) \dot{\gamma}_{ij}, \end{aligned} \quad (3)$$

The regularized Bingham model is used for the viscoplastic fluid. The strain scalar is as:

$$\dot{\gamma} = \sqrt{\frac{1}{2} \left[\sum_{i,j=1}^3 \dot{\gamma}_{ij}^2 \right]}. \quad (4)$$

The dimensionless groups in the Eqs. (1) to (4) are:

$$r_\mu = \frac{\mu_1}{\mu_2}, Re = \frac{\rho U h}{\mu_2}, Pe = \frac{U h}{D_m}, Bn = \frac{h}{\mu_2 U} \tau_y, \quad (5)$$

In which, ρ is the density of both fluids, h and U are characteristic length and velocity, respectively, μ is viscosity and D_m is the molecular diffusion coefficient.

Developed and steady flow considerations lead to a series of equations, which can be solved numerically by trial error method, to find the velocity profile and pressure drop in the developed region. This solution, which is achieved through a code in Wolfram language, is labeled as Semi-Analytic (SA) solution and used for the comparison with Spectral Element Method (SEM) results.

3- Numerical method

The rectangular domain is decomposed into a number of quadrilateral elements and then each element is transformed into a standard coordinate element using a mapping system. Every unknown value is approximated on this standard element through a modal expansion based on Legendre function. After linearization, all of the equations can be seen as a general Helmholtz equation. The spectral element discretization of the Helmholtz equation is the key part of the multi-step algorithm that is used to decouple the primitive variables, i.e. u , v , P and C . This algorithm is called velocity correction scheme and basically is used for the Navier-Stokes equations [3].

In the present work, the velocity correction scheme is developed in two steps. The first step is to upgrade the solver for Helmholtz equation with variable diffusion coefficient. This coefficient is used for the two phase flow with viscoplastic lubrication and is defined as:

$$D(\dot{\gamma}, C) = \left[\frac{C}{r_\mu} + \frac{1-C}{\mu(\dot{\gamma})} \right]^{-1}, \quad (6)$$

which is called the total apparent viscosity. The second step is to add a class to the basic solver for considering the concentration equation. These developments have been implemented on the Nektar++ framework to give the desired solver.

4- Results and Discussion

In Fig. 2, SEM, SA and the numerical results by Hormozi et

al. [4] for the outflow profile of x - velocity component, u , is presented. A good agreement between these three cases can be observed. The uniform distribution of velocity is related to the un-yielded region where the viscosity goes to infinity and strain approaches to zero.

The contour of total apparent viscosity is presented in Figure 3. The results of Figs. 2 and 3 are achieved for $Re = 5$, $Bn = 10$, $r_\mu = 1$ and $Y_i = 0.44$. The total apparent viscosity has the highest value in the un-yielded region.

5- Conclusions

A two-dimensional multi-layer core-annular flow with Bingham plastic fluid as the lubricant has been studied in this paper. The volume of fluid method has been used for the two phase flow modeling and the governing equation has been discretized spatially by the spectral element method. A high order multi-step splitting algorithm has been implemented on the Nektar++ framework. Moreover, a semi-analytic solution for the developed flow has been achieved by using a Wolfram code in Mathematica for comparing with the SEM one. The results show that the Bingham number is the most effective parameter on the pressure drop and un-yielded region thickness.

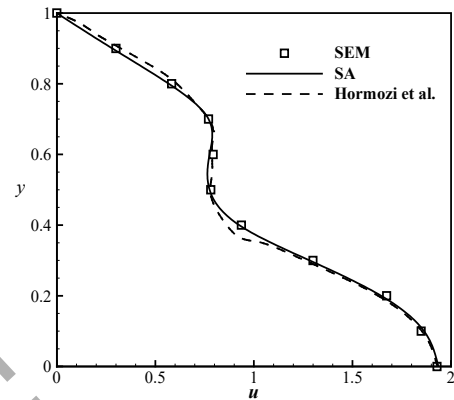


Fig. 2. Comparison of SEM and SA results of the channel outflow velocity profiles with the work of Hormozi et al. [4].

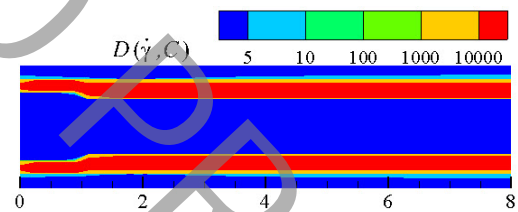


Fig. 3. SEM contour of total apparent viscosity

6- References

- [1] S. Ghosh, T.K. Mandal, G. Das, P.K. Das, Review of oil water core annular flow, Renewable and Sustainable Energy Reviews, 13(8) (2009) 1957-1965.
- [2] I.A. Frigaard, Super-stable parallel flows of multiple viscoplastic fluids, Journal of Non-Newtonian Fluid Mechanics, 100(1-3) (2001) 49-75.
- [3] G.E. Karniadakis, S.J. Sherwin, Spectral/hp Element Methods for CFD, Numerical Mathematics and Scientific Computation, Oxford University Press, New York, (1999).
- [4] S. Hormozi, K.W. Burchard, I.A. Frigaard, Multi-layer channel flows with yield stress fluids, Journal of Non-Newtonian Fluid Mechanics, 166(5-6) (2011) 262-278.