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Analysis of the Heat Transfer in a Multilayer Living Tissue Using the Galerkin Weighted Residuals Method

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ABSTRACT: In this paper, the thermal behavior of living biological tissue during electromagnetic radiation thermal therapy is investigated. While a large number of studies devoted to the Fourier and non-Fourier heat transfer in living tissue are available for different boundary conditions, less analytical and semi-analytical works exist on the heat transfer in the multilayers tissue. In the present study, semi-analytical Galerkin weighted residuals method is used to solve the dual-phase lag non-Fourier heat transfer equation in the multilayer tissue with a tumor placed in. The results show that considering a multilayer tissue with distinct thermophysical properties for each layer has a remarkable effect on the temperature distribution in the tissue, so that 2°C difference in tumor temperature after 1800 s is observed. The effect of the Vernot number on the temperature distribution shows that increasing the flux relaxation time results in reducing the temperature signal velocity and the tumor temperature. Lowering the skin surface temperature, decreases the high values of temperature and forces the maximum temperature region deeper into the tissue. Moreover, the reduction in the blood perfusion rate that occurs in the hypoxic tumors results in the increase of the tumors temperatures during the thermal therapy.

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1- Introduction

The therapeutic treatments based on the transfer of thermal energy into or out of the body, using nonionizing radiation have been called thermal therapy. One of the main concerns in thermal therapy is to control the tissue temperature in order to prevent the burn and damage of the healthy tissue. Many numerical and analytical studies on the bioheat equation models have been presented by researchers for various boundary conditions to find the temperature distribution in the tissue during the heat treatment process. Xu et al. [1] derived exact solutions for different bioheat equations for the single-layer skin tissue. In multilayer case they used finite difference method. Gupta et al. [2] used the Galerkin method taking Brenstein polynomials as basis function to solve one-dimensional Pennes bioheat equation. Kundu and Dewanjee [3] investigated the heat transfer in the single-layer skin tissue under different boundary conditions using the Laplace transform method. Askarizadeh and Ahmadikia [4] presented exact analytical solution of two-dimensional Pennes, thermal wave and Dual-Phase-Lag (DPL) bioheat transfer model. Verma et al. [5] studied two-dimensional heat transfer in a three-layered skin tissue. In the present study, the Galerkin weighted residuals method is employed to obtain a semi-analytical solution for the dual-phase lag bioheat equation in a multilayer tissue during moderate hyperthermia and the results are compared with the one-layer tissue modeling results. The effects of the temperature of the skin surface, the relaxation times, and the blood perfusion rate on the thermal response of each layer of tissue are investigated.

2- Methodologyg

The one-dimensional multilayer tissue is considered with thickness L=0.05m. The tissue which is initially at the body temperature is heated by an electromagnetic radiation with a 432 MHz antenna. The surface temperature of the skin is controlled by a cooling pad and therefore, the surface temperature of the skin remains constant during the treatment process. To investigate the effect of the cooling pad, the case without a cooling pad is also considered in this study. In the latter case, the convection boundary condition is considered on the surface of the skin. The dual-phase lag bioheat equation in the living tissues is:

$$\left(\rho_t c_t + \tau_q \omega_b \rho_b c_b\right)\frac{\partial T}{\partial t} + \tau_q \rho_t c_t \frac{\partial^2 T}{\partial t^2} = \nabla.\left(k\nabla T + k\tau_t \frac{\partial \nabla T}{\partial t}\right) + \tau_q\left(\frac{\partial Q_m}{\partial t} + \frac{\partial Q_{ext}}{\partial t}\right) + Q_m + Q_{ext} + \omega_b \rho_b c_b \left(T_a - T\right). \quad (1)$$

The initial conditions for Eq. (1) are:

$$T(x,0) = T_0, \qquad \frac{\partial T(x,0)}{\partial t} = 0, \quad (2)$$

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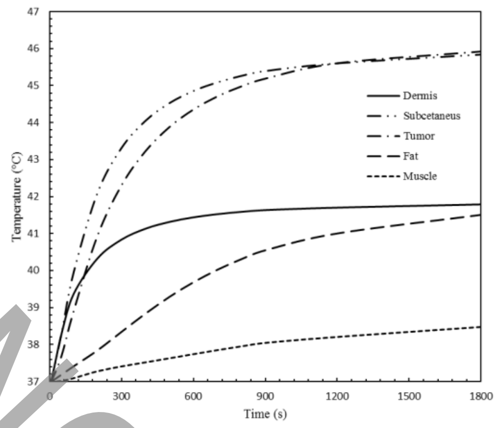

Fig. 1. Temperature variation with time in multilayer tissue

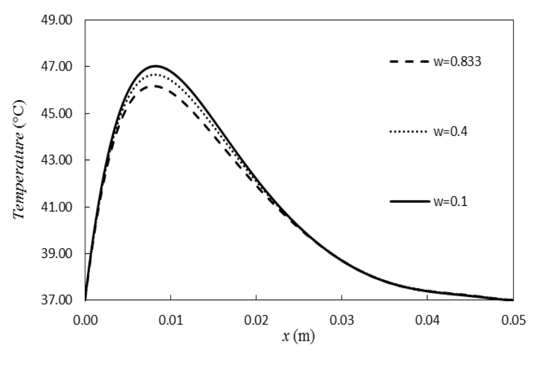

Fig. 2. Effect of the blood perfusion rate on tissue temperature distribution

The boundary conditions for the case of constant temperatures are:

$$T(0,t)=T_s, \tag{3}$$

$$T(l,t)=T_w. \tag{4}$$

and for the case of convection on the surface of the skin are as follows:

$$\frac{\partial T(0,t)}{\partial t}=h\left(T(0,t)-T_\infty\right), \quad T(l,t)=T_w. \tag{5}$$

where T_s is the skin surface temperature and T_w referred to the body temperature that is normally equal to 37 °C.

In this study, the Galerkin weighted residuals method is employed to obtain a semi-analytic, closed-form solution for this equation. In this method, the following approximate temperature profile is selected for the case of constant surface temperature:

$$T(x,t)=Ts\left(\frac{L-x}{L}\right)+Tw\left(\frac{x}{L}\right) +x(L-x)\sum_{i=0}^{n}C_i(t)N_i(x). \tag{6}$$

The approximate temperature profile for convection boundary condition is considered as:

$$T(x,t)=Tw+(L-x)\sum_{i=0}^{n}C_i(t)N_i(x). \tag{7}$$

where the Ni (x) is given by:

$$N_i(x)=x^i \ ,0\le i\le n \tag{8}$$

and Ci(t), i=0-n are the unknown coefficients. Substituting the appropriate temperature profile (Eq. (8) and (9)) into Eq. (1) results in a residual R. The unknown coefficients Ci (t) are obtained by equating the weighted integral of the residual to zero.

$$\int_0^L RW_i(x)\,dx=0. \tag{9}$$

The weighting functions are Wi(x)=x(L-x)Ni(x), for the constant boundary conditions and Wi(x)=(L-x)Ni(x), for the convection boundary conditions. As it is observed from Eq. (8), the selected temperature profile satisfies the boundary conditions at the surface and the end of the tissue for all values of the unknown coefficients. In the case of convection on the skin surface, the weak form of the resulting integration of the Eq. (11) should be derived to apply boundary conditions (6). Applying the Galerkin weighted residuals methods and simplifying the results, yields a system of ordinary differential equations for the unknown coefficients Ci (t), i=0-n. The initial conditions for these equations are obtained by applying the Galerkin weighted residuals method to the initial conditions (2) and (3). The system of ordinary differential equations is solved using the fourth-order Runge-Kutta method.

3- Results and Discussion

Fig. 1 shows the temperature distribution along with the tissue at different times for multilayer tissue.

As it is observed from this figure, after 30 minutes of applying the electromagnetic radiation, the temperature of the tumor reaches 46°C for multilayer tissue. For one-layer tissue, the temperature of the analogous position reaches 44°C. This difference arises due to different thermophysical properties of the different layers of the tissue. The effect of multilayer modeling of the tissue is more prominent in the dermis, subcutaneous and tumor layers. The high blood perfusion rate of the muscle layer causes lower temperatures in this region. The low thermal conductivity of the fat layer prevents heat diffusion and leads to higher temperatures in tumor region. Fig. 2 shows that the higher blood perfusion rate of the tumor predicts the lower temperature because the faster blood flow leads to the stronger convection heat loss through the blood.

4- Conclusions

The Galerkin weighted residuals method is used to obtain semi-analytical solutions for dual-phase lag equation in a multilayer living tissue during moderate hyperthermia. Two different boundary conditions, the constant temperature, and convection are considered. The results show that multilayer modeling of the tissue has a remarkable effect on the temperature of the different layers of the tissue, whereas 2°C difference is observed in the temperature of the tumor after 1800 s of applying the electromagnetic radiation. The skin temperature can reach as high as 62°C. Increasing the flux relaxation time leads to decrease the thermal signal through the tissue. Increasing the blood perfusion rate decreases the temperature of the tumor. Increasing the flux relaxation time leads to decrease the thermal signal through the tissue. Increasing the blood perfusion rate decreases the temperature of the tumor.

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