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The Effect of Curvature of Microbeam and Electrode on the Snap-Through and Pull-In Instabilities

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ABSTRACT: Due to the pull-in instability, the sustainability of micro-electro-mechanical systems is vulnerable. One of the proposed mechanism to improve the stability of these systems is the use of curved microbeams. The curvature causes the snap-through phenomenon by which the microbeam moves to its second stable position. Despite the advantages of snap-through, sometimes it leads to unstable conditions. In order to use the merits of curved structure and avoid the snap-through effect, in the present study, the performance of a structure composed of curved electrode is investigated. By assuming the Euler-Bernoulli beam theory and based on the modified couple stress theory, the governing equation is obtained by using Hamilton's principle. This equation is converted to a nonlinear ordinary differential equation by using the reduced-order model based on Galerkin procedure. The numerical solution is formulated and obtained by using the MATLAB software. The performance of the systems composed of curved microbeam and curved electrode are compared with each other, as well as with the systems made of straight elements. The results show that in cases where snap-through may cause unstable conditions, the use of curved electrode can result in more sustainable behavior in a wider range of position and voltage levels.

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1- Introduction

Curved microbeams are one of the most widely proposed mechanisms for controlling the instability of Micro-Electro-Mechanical Systems (MEMS). They may exhibit different behaviors, depending on the interaction between mechanical and electrical forces. One of these behaviors is the transmission between two stable states with a large domain, which is called snap-through [1]. Several results [2, 3] studying the possibilities of snap-through or pull-in depending on the level of electrostatic load and microbeam curvature confirm that sometimes the snap-through may result in the instability of the structure. So, if we can use the advantages of the curvature of the structure, and avoid the snap-through phenomenon, the structure behavior will improve. For this purpose, in present study, the behavior of the system composed of curved electrode has been compared with the systems composed of straight and curved microbeams.

2- Methodology

As shown in Fig. 1, generally the schematic of an electro-mechanic system may be composed of a curved microbeam in the vicinity of a curved electrode.

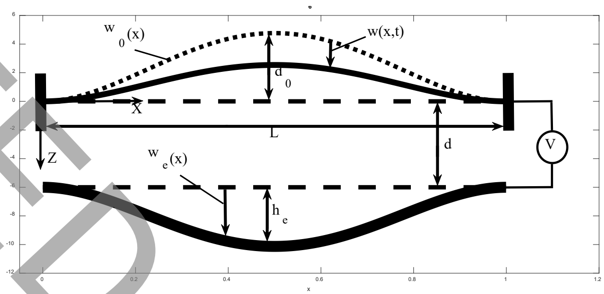

Fig. 1. Schematic model of the electromechanical system

In Fig. 1, $w_0(x)$ is the initial curvature of the microbeam, and is written as [4]:

$$w_0(x) = -\frac{d_0}{2}(1-\cos(\frac{2\pi x}{L})) \quad (1)$$

where d_0 is the maximum initial curvature of microbeam and L is its length. To have comparable shapes, the curvature of the electrode is selected similar to the curvature of microbeam as,

$$w_e(x) = \frac{h_e}{2}(1-\cos(\frac{2\pi x}{L})) \quad (2)$$

where h_e is the maximum curvature of the electrode.
Assuming shallow beam curvatures and Euler–Bernoulli beam model, by using Hamilton's principle and based on the modified coupled stress theory, the dimensionless equation of motion governing the normalized transverse deflection $W(\zeta,\tau)$ of the curved microbeam in the vicinity of the curved electrode is obtained as [4, 5]:

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$$(1+\eta)\frac{\partial^4 W(\zeta,\tau)}{\partial\zeta^4}+\frac{\partial^2 W(\zeta,\tau)}{\partial\tau^2}= \left\{\bar{P}_r+\alpha\int_0^1\left[\left(\frac{\partial W(\zeta,\tau)}{\partial\zeta}\right)^2+2\frac{\partial W(\zeta,\tau)}{\partial\zeta}\frac{dW_0(\zeta)}{d\zeta}\right]d\zeta\right\} \left\{\frac{\partial^2 W(\zeta,\tau)}{\partial\zeta^2}+\frac{d^2W_0(\zeta)}{d\zeta^2}\right\} +\beta\frac{V^2(\tau)}{(1+W_e(\zeta)-W_0(\zeta)-W(\zeta,\tau))^2} \quad (3)$$

where ζ and τ are the dimensionless position and time and V(τ) is the applied voltage, and the non-dimensional parameters η, $\bar{P}_r$, α, and β are defined as:

$$\eta=\frac{\mu A l^2}{EI},\bar{P}_r=\frac{P_r L^2}{EI},\alpha=6\left(\frac{d_{max}}{h}\right)^2,\beta=\frac{6\varepsilon L^4}{Eh^3 d_{max}^3} \quad (4)$$

here μ is the Lame's constant, A and I are the area and moment of inertia of the cross section, l is the material length scale parameter, E is the Young's modulus, P_r is the axial force, d_{max} is the maximum initial gap, h is the microbeam thickness and ε is the dielectric constant of the gap medium.

The boundary conditions are

$$W(0,\tau)=W(1,\tau)=\frac{\partial W(0,\tau)}{\partial\zeta}=\frac{\partial W(1,\tau)}{\partial\zeta}=0 \quad (5)$$

The initial conditions are assumed as follows:

$$W(\zeta,0)=\frac{\partial W(\zeta,0)}{\partial\tau}=0 \quad (6)$$

Due to the high level of non-linearity involved in Eq. (3), a closed-form solution for this equation cannot be found. Hence, an approximate solution based on the Galerkin weighted residual method will be developed. Relying on this method and assuming quasi-static conditions, the pull-in voltage and pull-in location of the system can be obtained. In this case, the deflection of microbeam based on one-mode solution is expressed as,

$$W(\zeta)=a\psi(\zeta) \quad (7)$$

Here, ψ(ζ) is normalized such that the parameter a describes the mid-point deflection of the microbeam. ψ(ζ) is the first linear and un-damped mode-shape of the un-deformed microbeam which is determined as [6]

$$\psi(\zeta)=0.6297\{\cosh(4.73\zeta)-\cos(4.73\zeta) -0.98[\sinh(4.73\zeta)-\sin(4.73\zeta)]\} \quad (8)$$

By setting the time derivative equal to zero in Eq. (3) and substituting Eq. (7) into the resulting equation, one obtains,

$$(1+\eta)a\frac{d^4\psi(\zeta)}{d\zeta^4}=\left\{\bar{P}_r+\alpha\int_0^1\left[a^2\left(\frac{d\psi(\zeta)}{d\zeta}\right)^2 +2a\frac{d\psi(\zeta)}{d\zeta}\frac{dW_0(\zeta)}{d\zeta}\right]d\zeta\right\}\left\{a\frac{d^2\psi(\zeta)}{d\zeta^2}+\frac{d^2W_0(\zeta)}{d\zeta^2}\right\} +\beta\frac{V_{DC}^2}{(1+W_e(\zeta)-W_0(\zeta)-a\psi(\zeta))^2} \quad (9)$$

Using MATLAB software, the numerical solution of Eq. (9) would specify the stability behavior of the system. Accordingly, once the gradient of voltage relative to deflection diminishes, the instability would happen.

3- Results and Discussion

By solving Eq. (9), the behavior of the systems made of each one of the straight or curved microbeams and the curved electrode is obtained. Fig. 2 shows an example of the results for the large ratio of curvature to gap.

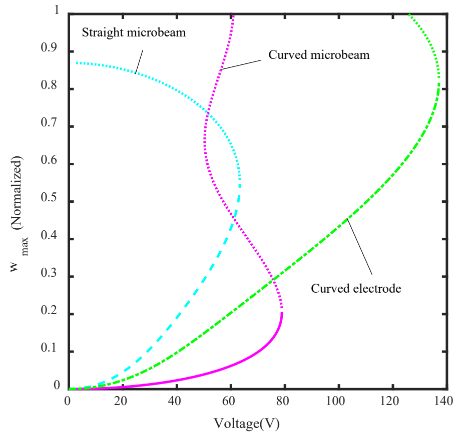

Fig. 2. Comparison of the non-dimensional deflection of different systems for the large ratio of curvature to the gap

In Fig. 2, the dotted point represents the unstable solution. Therefore, the curved microbeam just causes a slight increase in pull-in voltage and loses its stability in a much smaller position than the straight microbeam. In order to realize the effect of curvature size, instability characteristics of each system are calculated in different sizes of maximum gap, assuming h_e=3.5μm. These results are plotted in Figures 3 and 4.

Fig. 3 shows that the position of snap-through deserves the smallest size. Therefore, if the snap-through causes instability, the instability will start in a smaller position in comparison with other systems. In other words, to have the same instability characteristics, the curved electrode system must have smaller dimensions.

When the snap-through voltage is greater than the pull-in voltage, snap-through determines the instability conditions. Hence, the variation of instability voltages has to be calculated. Fig. 4 shows the variation of snap-through and pull-in voltage for different sizes of the maximum gap in the systems.

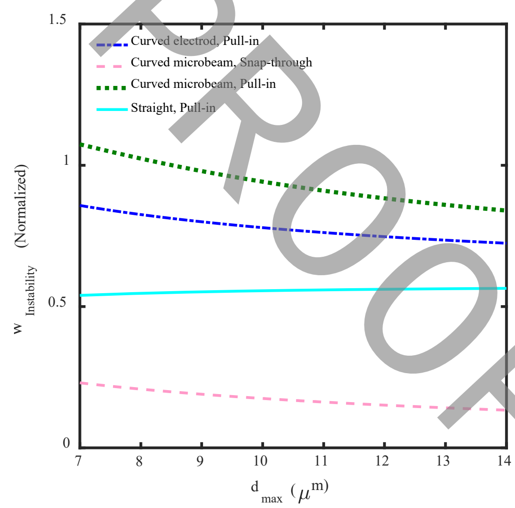

Fig. 3. Variation of instability positions for different sizes of the maximum gap, assuming h_e=3.5 μm

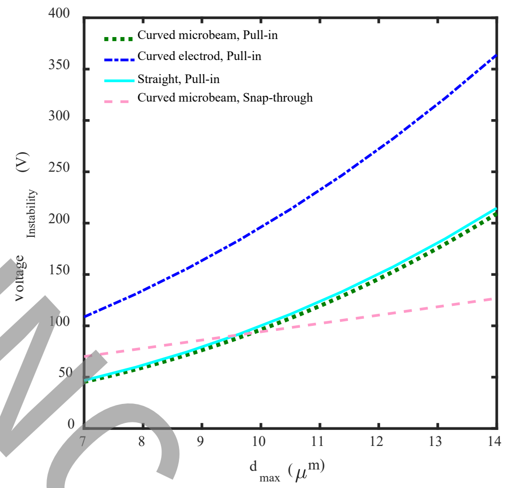

Fig. 4. Variation of instability voltages for different sizes of the maximum gap, assuming he=3.5 µm

Fig. 4 shows that the threshold endurance voltage of a system composed of a curved electrode is always larger than two other structures appointed in the figure. Therefore, by using a small structure composed of the curved electrode, one can achieve the same results as the greater systems composed of straight electrode, i.e. straight micro-beam or curved microbeam.

4- Conclusions

The aforementioned results on the instability of curved microbeams show that sometimes, snap-through is the main cause of structural instability. That is, the curvature of microbeam has negative effect on its stability behavior and causes pull-in in the smaller position comparing the straight microbeams. The results confirm that if snap-through is the main cause of instability in the curved microbeam, using a system composed of a curved electrode with the same dimensions would lead to the higher stable position and voltage levels.

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