



## The Effect of Dislocation Density on Forming Limit Curve

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**ABSTRACT:** One of the most important and widely used tools to predict the behavior of sheets is the forming limit diagram. The Marciniak-Kuczynski model is one of the prediction methods, which can be combined with the phenomenological or the crystal plasticity equations to achieve the desired results. In this research, to predict the forming limit diagram, the direct combination of the Marciniak-Kuczynski method with the crystal plasticity has been applied. The direct method is chosen due to the particular state of the mathematical equations associated with forming limit diagram. In this study a face-centered cubic polycrystalline metal has been used here, so, the Taylor method for the polycrystals can be used. Although this method ignores the interactions between the crystals to describe plasticity, it can also reduce the computational cost by simplifying the strain uniformity theory. In this study, polycrystal plasticity and dislocation methods have been merged in a new way. Only the hardening process is modeled based on the dislocation density and its modifications, and the entire analysis is based on the rate-dependent crystal plasticity. For the first time, the forming limit diagram is plotted to take into account the effect of dislocation density, and the results show that considering the effect of the dislocation density on the shear strength changes, the forming limit diagram formulation becomes nearer to the experimental values.

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### 1. Introduction

The crystal plasticity theory is able to detect micro-mechanisms of macroscopic phenomena in plastic deformation considering the slip in single crystals or single grains.

Moreover, analyzing the slip strength during plastic deformation and suggesting an efficient model is one of the most important and difficult issues of the crystal plasticity theory. The activity of each slip system is affected by its own hardening and hardening of other systems. Work hardening models are classified as a physical theory of crystal plasticity and mathematical one. The first mathematical model of work hardening was presented by Taylor [1]. In physical models, the work hardening is based on the variables of the microstructure of the crystal, such as the Stacking Fault Energy (SFE) and the dislocation density in the slip systems. Orowan, Polanyi, and Taylor almost simultaneously realized that plastic deformation can be explained theoretically by means of dislocations.

Taylor [1] found that the (atomic) flow stress is proportional to the square root of dislocation density [2]. Successful efforts have been made by Friedel [2] and Nabarro [3].

Although individual dislocation techniques have been investigated thoroughly, describing the behavior of the group of dislocation without the aid of computer modeling

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is challenging. Moreover, computer simulation examines the participation of various mechanisms by eliminating other factors.

Recent studies have shown that two-dimensional discrete dislocation dynamics simulation can predict similar results with 3-Dimensional (3D) simulation in some cases [4, 5]. Both types of simulations have been used to study a wide range of phenomena, including plastic flow in thin sheets of Davoudi et al. [6]; Bauschinger effect, Shishvan et al. [7]; Danas and Deshpande [8], Davoudi et al. [9]; and the effect of dislocation acceleration [10].

Although many dislocation evolution equations have been developed [11], Nix and Lee [11] have found that for bulk materials, there are several ways to use a discrete method to describe a dislocation evolution. Askari [12] suggested a continuous model for dislocation evolution in thin films with several crystals and used a discrete model to validate the model.

In this paper, it is aimed to predict the forming limits of the aluminum alloy using the hardening model based on the dislocation density and its combination with the crystal plasticity. According to the considering poly-crystal, the Taylor model is used.

### 2. Methodology

In this study, instead of using phenomenological models, an internal state variable model which is complementary to



the work of Li et al. [13] is developed based on dislocation activities in each slip system in Faced-Centered Cubic (FCC) material.

Threshold stresses in any slip system have a primary contribution to the critical resolved shear stress, grain size, or hardening of the Hall Patch effect and hardening of the forest dislocation; namely:

$$\tau^\alpha = \tau_0^\alpha + \tau_{HP}^\alpha + \lambda\mu b \sqrt{\Omega^{\beta\alpha} \rho^\beta} \quad (1)$$

The dislocation accumulation during plastic slip is evaluated by an ordinary differential equation. This equation can be rewritten as follows:

$$\frac{d\rho}{dt} = |\dot{\gamma}| (k_s \sqrt{\rho} - k_r \rho) \quad (2)$$

On the right side of Eq. (2),  $K_s$  indicates dislocation storage and  $K_r$  represents a dynamic recovery. These values are according to Ref. [14] and the references therein.

By referring to the relations of crystal plasticity Ref. [15] the main formulas for the solving algorithm are presented as follows:

The strain increment in the system  $\alpha$  and the linear relationship for its determination during the time interval  $\Delta t$  are defined according to Eqs. (3) and (4).

$$\Delta\gamma^\alpha = \gamma_{t+\Delta t}^\alpha - \gamma_t^\alpha \quad (3)$$

$$\Delta\gamma^\alpha = \left\{ (1-\theta)\dot{\gamma}_t^\alpha + \theta\dot{\gamma}_{t+\Delta t}^\alpha \right\} \Delta t \quad (4)$$

where, the subscript t represents the time and  $0 \leq \theta \leq 1$ .

According to the rate depends on the shear stress and shear strength in the slip system. Thus, using the Taylor expansion, one can write:

$$\dot{\gamma}_{t+\Delta t}^\alpha = \dot{\gamma}_t^\alpha + \frac{\partial \dot{\gamma}^\alpha}{\partial \tau^\alpha} \Delta \tau^\alpha + \frac{\partial \dot{\gamma}^\alpha}{\partial \tau_{cr}^\alpha} \Delta \tau_{cr}^\alpha \quad (5)$$

In this study, according to Eq. (1), the size of grains, the size of Burgers vector and the interaction matrix in shear strength changes are assumed constant during deformation. Therefore, the dislocation density is the only residual factor that is effective in the change of shear strength. Density variations are obtained through solving Eq. (2) which is a nonlinear Bernoulli differential equation.

$$\rho = \left( \frac{\chi}{\phi} + Ce^{-\frac{\delta t}{2}} \right)^2 \quad (6)$$

where  $\phi = |\dot{\gamma}| k_r$ ,  $\chi = |\dot{\gamma}| k_s$  and constant C is determined according to the initial value  $\rho$ . The initial value of dislocation density for all graphs is 107 cm<sup>-2</sup>, except when the density is mentioned. Finally, the equation is obtained for

different values of  $T_{cr}$  and Eq. (1) can be updated by putting this answer.

$$\tau^\alpha = \tau_0^\alpha + \tau_{HP}^\alpha + \lambda\mu b \sqrt{\Omega^{\beta\alpha} \left( \frac{\chi}{\phi} + Ce^{-\frac{\delta t}{2}} \right)^2} \quad (7)$$

### 3. Results and Discussion

A set of 65 crystals or grains with a random distribution is considered. The difference between the current solutions and the previous trend is in the use of two theories of slip resistance based on the total density of dislocations and the theory of variations in the density of statistical dislocations. To validate the results, the results are compared with the results of the Taylor method, the results of the self-consistent method proposed by Serenelli et al [16] and the experimental results. We define our method with TD (Taylor Dislocation) symbol. As shown in Fig. 1, experimental results are shown with a square mark. The results on the left side of the graph are better predicted by the recent method and the Taylor method and the self-consistent method in this area fails to provide the correct results. In the right-hand side of the graph, the results of the Self-Consistent (SC) method are closer to the experimental results and the new TD diagrams are closer to the SC method than the Taylor method. This indicates increasing the accuracy of our method compared to Taylor method. In fact, the latter method provides a significant improvement in Taylor predictions and in terms of computational cost, it is much better than self-consistent method. However, due to simplifications such as eliminating the effect of the density of geometry necessary dislocations, the results are far from experiments but they are acceptable compared to the two existing Taylor and self-consistent methods.

In Fig. 2, the experimental results of AA5182-O are marked with a square mark. On the left side of this diagram, the results of the TD method are closer to the test results with two other methods. In fact, the self-consistent method,

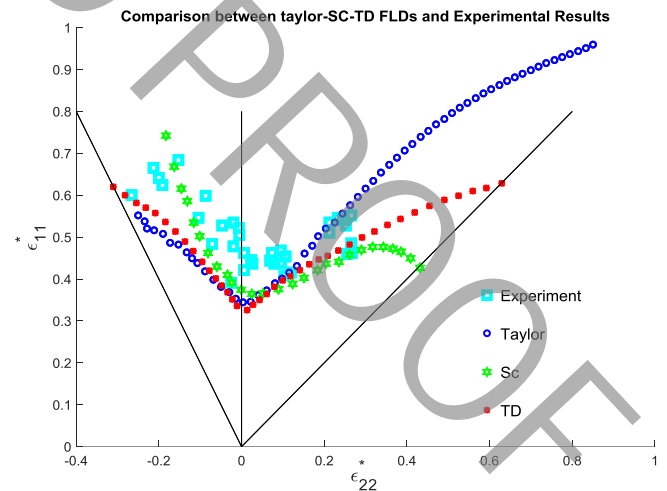
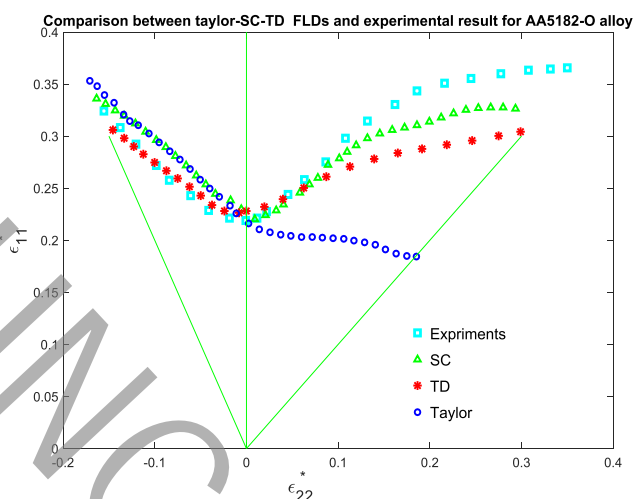


Fig. 1. Comparison between the forming limit diagrams obtained by Taylor-model, the self-consistent method of Serenelli et al. [16] and Taylor method based on dislocation



**Fig. 2. Comparison of forming limit diagrams obtained by the self-consistent method, Taylor method and proposed method with the experimental results**

despite being more precise in many cases, does not predict the exact location of the FLD on the left side.

#### 4. Conclusions

In this paper, a model with the combination of the m-k method with Taylor's polycrystalline analysis and the effects of dislocations and their changes during deformation is presented. Using this model, the aluminum forming limit diagrams are predicted. By comparing the obtained results of predicting the forming limit diagram with the experimental results and also the results of the Taylor method and the self-consistent method, the following findings are achieved.

- The TD method, by considering the effect of statistical dislocation density, attempts to improve the Taylor method for predicting the behavior of polycrystals, which has greatly improved the results of the Taylor method.
- The left side of the graph of the prediction of Taylor-method is better than the self-consistent method. The TD method by considering the dislocation density has improved the Taylor method either on the right or on the left side of the diagram. Consequently, the left-hand side of the graphs is very close to the experimental results. The right side of the graphs is also close to the self-consistent graphs.
- The accuracy of the results enhances as the number of grains and the number of grain orientations increase.

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