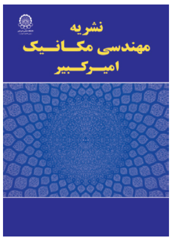

Amirkabir Journal of Mechanical Engineering

Amirkabir J. Mech. Eng., 53(1) (2020) 1-3
DOI:

Estimation of Stress Concentration Factor of Elliptical Cutout in the Composite Sheets under Tensile Load

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ABSTRACT: In this paper, we try to use a multiple linear regression method to explicitly describe the explicit relation between the stress concentration coefficient of plates containing elliptic openings in terms of mechanical properties and the angle of rotation of openness. First, the stress concentration coefficient for a lot of composites was calculated by the analytical method based on the mixed variable method and using different values of the mechanical properties of the composites. Using the obtained data, using the regression method multiple linear the explicit relation was used to estimate the stress concentration coefficient in unbounded composite plates containing elliptic openings under tensile load. It is important to note that several factors affect the concentration of tension around openness. Therefore, by correct selection of these parameters, it can be significantly reduced the stress concentration and increased the structural strength. One of these factors is the angle of rotation of openness, which is discussed in this article. In addition to the ease of use, the proposed relationship, by eliminating complex and complex analytical solutions, saves time and allows the designer to obtain the desired parameters to achieve the desired stress. The results showed that the regression model was able to estimate the coefficient of stress concentration with an error of less than 1 percent.

Review History:

Received:
Revised:
Accepted:
Available Online:

Keywords:

Stress concentration factor
elliptical hole
multiple linear regression
angle of rotation of cutout
complex variable method

1- Introduction

Nowadays, the importance of using composites and the use of these materials in various industries is not overlooked. In many cases, the designer is bound to create a cutout in the plan for an optimal design. The existence of these geometric discontinuities in the body creates a phenomenon called stress concentration factor.

Muskhelishvili [1] was able to present a complex variable method for isotropic elastic materials. He limited the determination of stress concentration factor in a plate containing cutout, with the calculation of two holomorphic analytic functions. Savin [2] examined stress concentration in infinite plates for isotropic and anisotropic materials. In identical materials, he presented stress distribution surrounding the cutout with various forms, and in anisotropic materials, he analyzed only the circular and elliptical cutouts. In his researches, he used the Muskhelishvili's complex variable method. Lekhnitskii [3] by developing Muskhelishvili's complex variable method, was able to provide a solution for different cutouts in an anisotropic infinite plate. Abbasnia et al. [4] presented a new equation for estimating the coefficient of stress concentration of circular cutout in the orthotropic plate. They showed that their model was able to predict the stress concentration factor with a maximum error of 1%.

2- Methodology

Composite plates studied in this paper, plates containing elliptical cutout are considered under axial loading and unlimited plates. The cutout is located at the center of the plate (Fig. 1).

According to the Lekhnitskii elasticity theory in anisotropic bodies, the stress function calculation is depended on the calculation of unknown coefficients. These coefficients are calculated according to the boundary conditions around cutouts. The stress function is presented in infinite anisotropic infinite plates under inplane loading containing circular and elliptical cutouts by Savin [2] and Lekhnitskii [3].

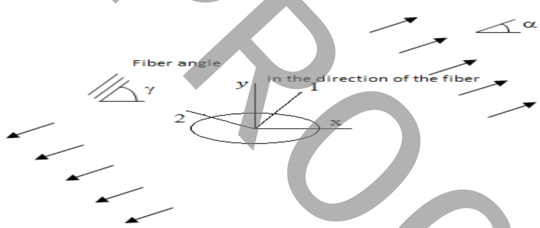

Fig. 1. Schematic diagram of an infinite plate with elliptical cutout under uniaxial tension.

If $F(x,y)$ is the stress function of the present problem; by substituting it in the equilibrium equations, the fourth order differential equation is obtained:

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$$R_{22}\frac{\partial^4 F}{\partial x^4}-2R_{26}\frac{\partial^4 F}{\partial x^3\partial y}+(2R_{12}+R_{66})\frac{\partial^4 F}{\partial x^2\partial y^2}-2R_{16}\frac{\partial^4 F}{\partial x\partial y^3}+R_{11}\frac{\partial^4 F}{\partial y^4}=0 \quad (1)$$

In fact, the Eq. (1) is the equilibrium equation for an anisotropic material, where the stress function $F(x,y)$ and R_{ij} are the coefficient of the reduced compliance matrix. Base on roots of the characteristic equation, the general expression of the F stress function is as the Eq. (2):

$$F(x,y)=2\,\mathrm{Re}[f_1(z_1)+f_2(z_2)] \quad (2)$$

where f_1 and f_2 are arbitrary functions of the complex variable $z_k=x+\mu_k y$ for $k=1,2$. By employing this approach, the problem is reduced to finding two complex functions f_1 and f_2 such that the boundary conditions satisfy the edge of the cutout. Therefore, stresses are determined by the Eq. (3).

$$\begin{aligned}\sigma_x&=2\,\mathrm{Re}[\mu_1^2 f_1''(z_1)+\mu_2^2 f_2''(z_2)]\\ \sigma_y&=2\,\mathrm{Re}[f_1''(z_1)+f_2''(z_2)]\\ \tau_{xy}&=-2\,\mathrm{Re}[\mu_1 f_1''(z_1)+\mu_2 f_2''(z_2)]\end{aligned} \quad (3)$$

According to the cutout border, it is better to provide mentioned stresses in the polar coordinate system. Because in this device, the shear and radial stresses ($\tau_{r\theta}=\sigma_r=0$) will be zero at the cutout boundary and the only remaining stress is circumferential stress (σ_θ).

As mentioned, the stress concentration factor ($\frac{\sigma_{\theta\max}}{\sigma_0}$) in plates with geometric defects is a function of five variables $E_1, E_2, G_{12}, \nu_{12}$ and β. The applied stress in the boundary of the σ_0 plate was considered equal to 1Pa. The stress concentration factor is a dimensionless variable, therefore, for dimension Lessing variables, were used different ratios of E_1, E_2, G_{12}.

Regression analysis was used to obtain mathematical relation. A quadratic model was used in accordance with Eq. (4). In this regard, the dependent variable (y) is equivalent to the stress concentration factor.

$$y=\beta_0+\sum_{i=1}^{m}\beta_i x_i+\sum_{i=1}^{m}\sum_{i=2}^{l}\beta_{ij}x_i x_j+\sum_{i=1}^{m}\beta_{ii}x_i^2+\varepsilon \quad (4)$$

Where β0 is the constant coefficient or the intercept, βi is the linear factor of the model or simple effects, βij is the factors of interactions of independent variables, βii is second power factors of independent variables and $\varepsilon\varepsilon$ is the model error.

Different regression models were used to predict the stress concentration coefficient. These models are linear model, two-factor interaction model, quadratic model, reduce quadratic model. These models can be expanded depending on the number of independent variables.

3- Results and Discussion

By choosing the variables $\frac{E_2}{E_1}$, $\frac{E_1}{G_{12}}$ and ν_{12} the lowest root mean square error and Mean Absolute Percentage Error (MAPE) can be obtained in estimating the stress concentration coefficient [4]. Therefore, in this research, the variables mentioned along with the angle of rotation of the cutout variable β are used to obtain the equation that calculates the stress concentration coefficient in plates containing elliptical cutouts. Model form, model defect, model variance analysis, survey evaluation, stability and generalizability of the regression model are discussed after the selection of variables.

The results of this study are compared with the finite element solving for graphite/epoxy material with mechanical properties of $E_1=181$, $E_2=10.3$, $G_{12}=17.7$ and $\nu_{12}=0.28$ at angles of rotation of cutout, the results of which are reported in the form of Table 1.

Table 1. Comparison of the calculated stress values in the present study and the solving finite element at different angles of rotation for the graphite/epoxy graphite containing elliptical cutout

β	Present Study	Finite Element	Difference %
25	5.4694	5.4363	0.62
85	12.4110	12.4359	0.20
90	12.4901	12.5010	0.08
155	5.4696	4.4350	0.64

Therefore, the results showed that the extracted model based on the regression method could be used to estimate the stress concentration coefficient of ellipticity cutout according to the mechanical properties of the material and the angle of rotation of cutout (Eq. (5)).

$$\begin{aligned}\ln(\frac{\sigma_\theta}{\sigma_0})&=0/01-0/70(\frac{E_2}{E_1})+0/01(\frac{E_1}{G_{12}})+0/15\upsilon_{12}\\ &+0/72\sin\beta+0/28(\frac{E_2}{E_1})^2-0/0001(\frac{E_1}{G_{12}})^2-0/21\upsilon_{12}{}^2\\ &-0/01\sin\beta^2+0/01(\frac{E_2}{G_{12}})+0/31(\frac{E_2}{E_1})(\upsilon_{12})\\ &+0/16(\frac{E_2}{E_1})(\sin\beta)+0/004(\frac{E_1}{G_{12}})(\upsilon_{12})\\ &-0/002(\frac{E_1}{G_{12}})(\sin\beta)+0/08(\upsilon_{12})(\sin\beta)\end{aligned} \quad (5)$$

4- Conclusions

In this research, using an analytical solution method based on the complex variable method, stress concentration for a large number of orthotropic plates under axial stress was calculated with elliptic cutouts; then, using multiple linear regression, an explicit relationship between the coefficient of stress concentration and mechanical properties for elliptical cutouts with orthotropic plate.

The evaluations proved the accuracy of the presented mathematical model. This relation is proposed for infinite plates that are under axial stress, which provides an explicit relation between the stress concentration coefficient and the mechanical properties of matter and the rotation angle of the cutout. In addition, the equality of the correlation coefficient between two actual and predicted data sets in the calibration stages of the model and test (equal to 0.98) proves the proposed solution to be reliable.

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