



2D Numerical Modeling of Compressible Two- Phase Flows Using Hyperbolic Two Pressure Two- Fluid Model

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ABSTRACT

Numerical modeling of compressible two-phase flow is a challenging and important subject in practical cases and research problems. In these problems, mutual effect of shock wave interaction creates a discontinuity in the fluid properties and interface of two fluids as a second discontinuity lead to some difficulties in the numerical approximations and estimating an accurate interface during the hydro-dynamical capturing process. The main objective of this research is accurate capturing of the interface and numerical study of shock wave during gas-gas and gas-liquid interface of two-phase flows. For these purposes , HLLC Riemann solver and Godunov numerical method was used for a hyperbolic two-pressure two-fluid model where programming was conducted in two-space dimensional with the second order accuracy. Various one and two-dimensional problems were simulated such as compression and expansion shock tubes, shock wave interaction with R22/air bubble, underwater explosion and hypersonic shock with $M=6$ interaction with a cylindrical water column. The numerical results obtained from this attempt exhibit very good agreement with the experimental results, as well as the previous numerical results presented by the other researchers based on the other numerical methods.

KEYWORDS

Two Phase Flow, Hyperbolic Two-Fluid Model, Compressible, Shock Wave, Godunov Numerical Scheme, And Interface.

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1- INTRODUCTION

Compressible multi-material flows and multiphase mixtures arise in many natural and industrial situations including bubble dynamics, shock wave interaction with material discontinuities, detonation of high energetic materials, hypervelocity impacts, cavitating flows, combustion systems. The models and numerical simulations presented, in recent literature, present different levels of accuracy and complexity. In general, these types of methods can be separated into two categories by how each one considers the interfaces:

1-Sharp interface method (SIM)

2-Diffuse interface method (DIM)

In the sharp interface methods, a special effort is made to find the right location of the interface and to treat the interface explicitly. In the second group of numerical methods, DIM, the interface is modeled as a numerically diffused zone (area). This is similar to capturing a discontinuity in gas dynamics. One model that is suitable for multiphase flow simulation is the reduced five-equation model, also known as the Kapila model [1-2]. There are two main problems in using this model. First, the mixture sound velocity at the interface has non-monotonic behavior. Second, the volume fraction equation has a non-conservative term [3]. This model consists of two mass conservation equations, one momentum conservation equation, one energy conservation equation in the conservative form and one volume fraction advection equation in the non-conservative form. In this paper, six equation two pressure model that is a kind of multiphase flow models was used. The six-equation model is obtained from the seven-equation model in the asymptotic limit of zero velocity relaxation time [3-4]. In this study, the HLLC Riemann solver is used for the numerical simulation of compressible two-phase flow. The main innovations of this paper are:

Using appropriate sound velocity with less non-monotonic behavior, the Wood sound relation is not applicable.

A suitable discretization of the advection equation. Preventing negative pressure during numerical calculation of cavitation zones due to strong rarefaction waves reflecting from free surfaces by adapting a suitable cavitation equation of state.

Numerical solution of the governing equations using the Godunov numerical method and the HLLC solver.

2- TWO FLUID MODEL AND NUMERICAL METHODES

The six-equation model without heat and mass transfer can be written as:

$$\frac{\partial \alpha}{\partial t} + \vec{u} \cdot \vec{\nabla} \alpha = \mu(P_i - P_r) \quad (1a)$$

$$\frac{\partial(\alpha_r \rho_r)}{\partial t} + \nabla \cdot (\rho_r \alpha_r \vec{u}) = \cdot \quad (1b)$$

$$\frac{\partial(\alpha_i \rho_i)}{\partial t} + \nabla \cdot (\rho_i \alpha_i \vec{u}) = \cdot \quad (1c)$$

$$\frac{\partial(\rho \vec{u})}{\partial t} + \nabla \cdot (\rho \vec{v} \otimes \vec{u}) + \vec{\nabla} P = 0 \quad (1d)$$

$$\begin{aligned} \frac{\partial \alpha_i \rho_i e_i}{\partial t} + \nabla \cdot (\alpha_i \rho_i e_i \vec{u}) + \alpha_i P_i \nabla \cdot \vec{u} \\ = -P_i \mu(P_i - P_r) \end{aligned} \quad (1e)$$

$$\begin{aligned} \frac{\partial \alpha_r \rho_r e_r}{\partial t} + \nabla \cdot (\alpha_r \rho_r e_r \vec{u}) + \alpha_r P_r \nabla \cdot \vec{u} \\ = +P_r \mu(P_i - P_r) \end{aligned} \quad (1f)$$

$$\frac{\partial \rho E}{\partial t} + \nabla \cdot (\rho E \vec{u} + P \vec{u}) = \cdot \quad (1g)$$

Where α, ρ, u, P, E, e are the volume fraction, density, velocity, pressure, total energy and internal energy, respectively.

In the present work, the stiffened-gas equation of state (SGS) is used. In this article, the Godunov numerical method was applied using the HLLC Riemann solver [5]. To achieve the second-order accuracy, the MUSCL method was used [5]. This method is conducted in three steps, which include data reconstruction, evolution and solving of the Riemann problem. A structured grid is used in the present work.

3- RESULTS

Three standard test cases that include interface were considered in this article:

- 1- Shock and R22 bubble interaction (figs (1),(2)).
- 2- Shock and water column interaction.
- 3- Under water explosion problem.

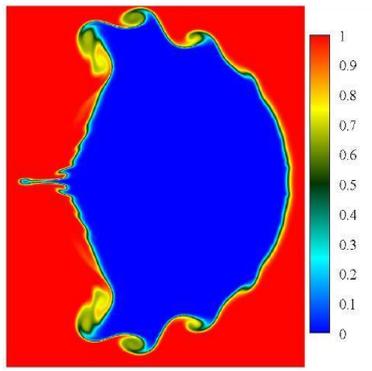


Fig. 1. Enlarged view of the interface topology at 432 μsec, instabilities occurring at the interface, for a Mach 1.22 shock interacting with R22 cylindrical bubble

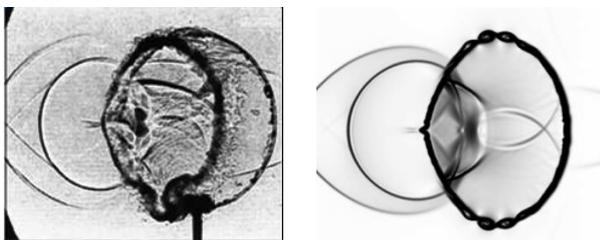


Fig.2. left: experimental results of shock bubble interaction [6] at $t = 247 \mu\text{sec}$, right: present numerical results.

In the Shock and R22 bubble interaction test case, a planar shock wave is moving in the air with $M_s = 1.22$ and collides with a cylindrical R22 bubble. R22 has a higher density and a lower ratio of specific heats than air, resulting in a speed of sound about two times lower than that of air. The lower speed of sound causes the shock wave inside the bubble, the refracted shock wave, to be delayed behind the incoming shock, Figs(1-2). Furthermore, the velocity of air behind the incident shock wave is greater than the velocity of air behind the transmitted shock wave inside the bubble. This velocity difference introduces a counter-clockwise shear tension on the surface of the bubble, which later leads to Kelvin-Helmholtz (K-H) instability. In addition, the accelerated contact surface is deformed, and Richtmyer-Meshkov (R-M) instability is observed due to the interaction of the shock system with the curved front of the bubble. The unstable interface eventually rolls up to form vortices and fragments of vortices on the interface. The results obtained from the numerical simulation exhibit no oscillation and have enough resolution to simulate the Richtmyer–Meshkov (R-M) and Kelvin–Helmholtz (K-H) instabilities.

4- CONCLUSION

In this paper, the capability and effectiveness of the numerical method were investigated for two different two-phase gas-gas and gas-liquid flows in the presence of shock waves. The numerical results of 2D simulations were accurate with no numerical oscillation. These results are in good agreement with the experimental results and previous numerical results obtained through more sophisticated numerical methods.

5- REFERENCES

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