



## Nonlinear Vibration and Stability Analysis of Thermally Postbuckled Double-Layered Graphene Sheet

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**ABSTRACT:** In the present research, the vibration behavior is presented for a thermally postbuckled double layered graphene sheet. For this purpose, the graphene sheet is modeled as a non-classical orthotropic plate. The formulations are based on the Kirchhoff's plate theory, and the von Karman-type nonlinearity is considered in the strain-displacement relations. Eringen's nonlocal elasticity theory is employed to incorporate the size effects. The thermal effects, van der Waals forces between layers and chirality are also included and the material properties are assumed to be temperature-dependent. A semi analytical solution is obtained using multiple time scales method. The effects of variation of the small scale parameter to the natural frequencies, deflections and response curve of double layered graphene sheet are analyzed and the numerical results are obtained from the nonlocal plate model. Numerical results are compared with those of similar researches. Effects of various parameters on the postbuckled vibration of graphene sheet in thermal environments such as the scale parameter, length, and thermal load are presented. The stability of vibration modes around a buckled configuration is investigated. The results show that the scale parameter and thermal changes have very important roles on the nonlinear vibrational behavior of the nano scale buckled structures.

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### 1- Introduction

Graphene structures with different shapes and configurations have superior properties and they are introduced as one of the fundamental carbon forms and graphene is the base of many other configurations such as graphite, carbon nanotubes, and fullerenes; therefore, studying single and double layer graphene sheets is very important in nanoscale studies [1]. In nanoscale plate problems, solutions due to classical theories such as Kirchhoff's plate theory and shear deformation plate theory usually have remarkable errors because in these theories, the relation between stress and strain is point wise and the size effects are not considered [2]. In nonlocal elasticity theory, the stress at a reference point is assumed to be a function of the strain field at every point in the body and the effects of scale are considered by a new quantity called scale parameter [3]. Based on nonlocal elasticity theory, size effects are taken into account by the integration of a scale parameter into classical continuum models [2, 3].

The thermal effect has one of the most important roles on the vibration behaviors of structures in macro and also nanoscales such as nanotubes and nanoplates. If the temperature of the plate is raised or lowered it expands or contracts, respectively. Within a certain temperature change, such expansion or contraction, for most structural materials, is directly proportional to the change in temperature. When a free plate made of homogeneous isotropic material is heated

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uniformly, there appear normal strains but no thermal stresses [4]. But for the case of graphene, considering thermal effects is very important. Researchers showed that the thermal effects on the mechanical behaviors of the carbon nanotubes are obvious [5].

In the present research, a nonlocal plate model to study the nonlinear vibration behavior of postbuckled Double-Layered Graphene Sheet (DLGS) is proposed. The governing equations are based on classical thin plate theory with a von Karman-type of nonlinearity and containing small scale effects. The thermal effects are also included and the material properties are assumed to be orthotropic and temperature-dependent. The effects of various parameters to the natural frequencies, deflections and also chirality of postbuckled DLGS are analyzed, and the numerical results are obtained from the nonlocal plate model. The numerical illustrations show nonlinear vibration response of postbuckled DLGS under fully clamped and different sets of thermal and environmental conditions. Also the stability of vibration modes around a buckled configuration is investigated.

### 2- Methodology

In this research, the vibration response of postbuckled DLGS has been investigated on the basis of a nonlocal plate model for fully clamped boundary condition. The major difference between present model and previous ones in literature is that the present solution includes the vibration and deflection



caused by thermal loading. For solving the postbuckling problem, Galerkin method is applied to the compatibility and equilibrium equations with the aid of airy stress function. Then, the system is solved by Newton-Raphson method. The complete derivation of nonlinear vibration of postbuckled DLGS is presented and the solution is introduced by solving a system of ordinary differential equations of order 2 by means of multiple time scale method. The efficiency and accuracy of the method are demonstrated by comparing the developed result with those available in literature. The effects of various parameters on the nonlinear vibrations of postbuckled DLGS are presented. Also the static stability is investigated using a method suggested by Nayfeh and Emam [6].

### 3- Results and Discussion

Effects of various physical parameters on the nonlinear vibration of postbuckled DLGS such as scale parameter, chirality, van der Waals (vdW) and temperature rise are investigated. Some of the numerical results are represented in the following Figures:

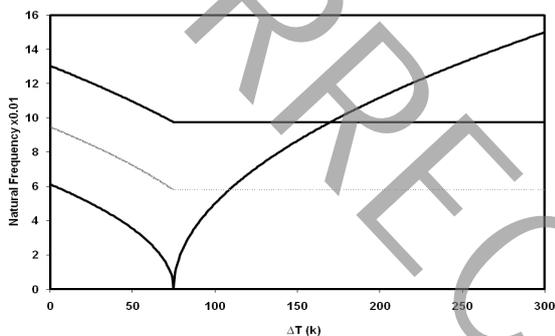


Fig. 1. Variation of the natural frequencies of vibration around the 1st buckled configuration of all sides clamped zigzag DLGS

In Fig. 1, the variation of the vibration frequencies around the first buckling mode with the temperature is presented. Solid lines show odd vibration modes and dotted lines indicate even ones.

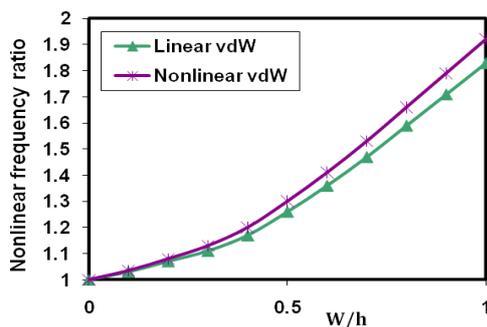


Fig. 2. The response curve for zigzag DLGS with linear and nonlinear vdW interaction

Effect of nonlinear vdW forces coefficient is shown in Fig. 2. It can be seen that the effect of nonlinear coefficient  $e_j$  is higher as the nonlinear frequency ratio is increased.

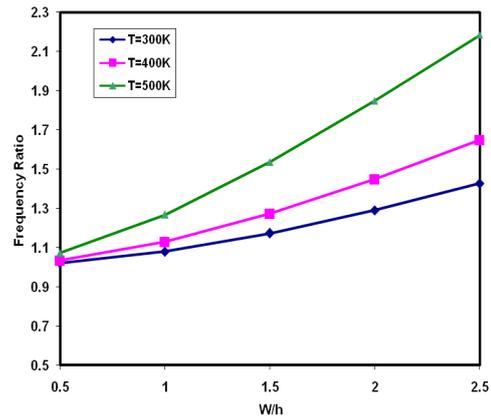


Fig. 3. Effect of temperature change on the frequency-amplitude curves of zigzag DLGS

The effect of temperature change on the response curves of DLGS under different thermal conditions is shown in Fig. 3. Results show that the increasing temperature reduces the natural frequencies by reducing the stiffness of nanoplate but increases the nonlinear to linear frequency ratios.

### 4- Conclusions

In this research, the vibration response of postbuckled DLGS has been investigated on the basis of a nonlocal plate model for fully clamped boundary condition. The main difference between present model and previous ones in literature is that the present solution includes the deflection and vibration caused by thermal effects. The solution of the problem is posed into two phases, one is the thermal postbuckling (static problem) and another one is the vibrations of large amplitude around the static response. In thermal postbuckling analysis, critical buckling temperature is calculated and stability of postbuckled modes is investigated by using linear vibration analysis. The complete derivation of nonlinear vibration of DLGS is presented and the solution is introduced by solving a system of ordinary differential equations of order 2 by means of multiple time scale method. The effect of chirality on the buckling temperature and vibration behavior of DLGS under similar thermal environmental and dimensional condition by considering two types of zigzag and armchair is analyzed. The numerical results showed that effect of chirality on vibration behavior is remarkable. Also results show that the armchair sheets will have lower natural frequencies than those of zigzag sheets when the two sheets have the same dimensional properties and thermal environmental condition. The stability analysis reveals that the first buckling mode is a stable mode; whereas, buckled configurations beyond the first buckling mode are found to be unstable. Effect of scale parameter on the different quantities such as natural frequency, critical temperature, and response curve is investigated. It is seen that as a result of employing the nonlocal theory, the beginning of instability is transferred to a lower temperature; consequently, taking into account the length scale parameter decreases the

flexural stiffness of the system. Also, increasing nonlocal parameter increases the nondimensional deflection of the plate. By increasing temperature, the natural frequencies have two opposing behaviors; decreasing before buckling state and increasing after buckling state. The results show that scale parameter and temperature change have a significant role in nonlinear vibration of postbuckled nanostructures.

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