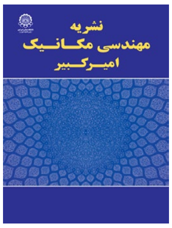

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Nonlinear Flapping-Torsional Free Vibration Analysis of Rotating Beams Considering the Coriolis Force

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ABSTRACT: The nonlinear free flapping-torsional vibration of rotating beams is investigated in this paper. The presented equations are based on the exact geometrical formulation in conjunction with the Cosserat theory for rods. The equations of motion are reduced to the flapping and torsional equations of motion for symmetric rectangular beams by neglecting the shear deformation. The governing equations are coupled to each other with the non-homogenous boundary conditions. By employing the direct method of multiple scales the effective nonlinearity coefficients of nonlinear natural frequencies are extracted. After validation of the current results, the effects of the rotating speed on the type and the value of the effective nonlinearity coefficient of natural frequencies are examined. The sign of the effective nonlinearity coefficient demonstrates the softening or hardening treatment of the corresponding nonlinear natural frequencies. It is concluded that ignoring the flapping-torsional coupling due to the Coriolis force, for odd modes makes some errors in the magnitude of effective nonlinearity but the type of nonlinearity is predicted correctly. On the other hand, in the even modes for average to high rotation speed in addition to incorrect estimation of the magnitude of effective nonlinearity the different type of nonlinearity is also predicted.

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1- Introduction

Rotating beams are in sight in many industries, including aerospace industry and wind, water and gas power plants. On the other hand, the sensitivity and high cost of the related structures demands their accurate modeling and precise prediction of their dynamics and vibration. Therefore, in this paper, their nonlinear flapping-torsional free vibration is investigated.

Da Silva and Hodges [1] analyzed the effect of different nonlinear terms including geometrical nonlinearity and the terms caused by aerodynamic forces on the stability of rotating blades. Valverde and Garcia-Vallejo [2] presented two different formulations using the absolute nodal coordinate formulation versus the exact geometrical formulation to analyze the stability of rotating beams. Arvin et al. [3] examined the nonlinear free vibration of flapping and longitudinal motions of rotating beams based on the exact geometrical formulation by the implementation of the direct Method of Multiple Scales (MMS). Arvin and Lacarbonara [4] developed the precise formulation for composite rotating blades by providing nonlinear constitutive relationships for composite materials. Arvin and Bakhtiari-Nejad [5] applied the MMS on the discrete motion equations of rotating Euler-Bernoulli beams to achieve the nonlinear natural frequencies and the corresponding nonlinear normal modes.

After the literature review, it is observed that in studies conducted so far, the nonlinear free vibration analysis of coupled flapping-torsional motions caused by the Coriolis force has not been investigated. Therefore, in this paper, taking into account the Coriolis force, the value and sign of the Effective Nonlinearity Coefficient (ENC) (indicating the softening or hardening of the nonlinear natural frequency) is evaluated for composite rotating beams with symmetric layup and rectangular cross-section.

2- The Composite Rotating Beam Modeling

A schematic of a multi-layer rotating beam which rotates by speed ω_R around axis $\boldsymbol{i}_1$ with length L, width b, thickness h and rotor radius d_3 is shown in Fig. 1(a). Two main coordinate systems are adopted to define the beam configurations; $\boldsymbol{e}_k$ -system for stress-free and $\boldsymbol{b}_k$ -system for the current configurations. A rotation tensor is determined to relate the two coordinate systems by employing an interface coordinate system $\boldsymbol{e}_k^{(1)}$ presented in Figs. 1(b) and 1(c). $\boldsymbol{r}(s,t)=s\boldsymbol{e}_3+\boldsymbol{u}(s,t)$ is the position vector of the mass center of an arbitrary point along the beam span in the current configuration in which $\boldsymbol{u}(s,t)$ is the displacement vector of the considered point at position s.

The governing equations are on the basis of the Cosserat theory for rods. The un-shear ability assumption is also adjusted. Due to the brevity all the mathematical procedures are dropped and the readers are invited to see references [3,4] for more illustrations.

3- Solution Process

The direct MMS is applied on the dimensionless governing equations to derive the nonlinear natural frequency and the

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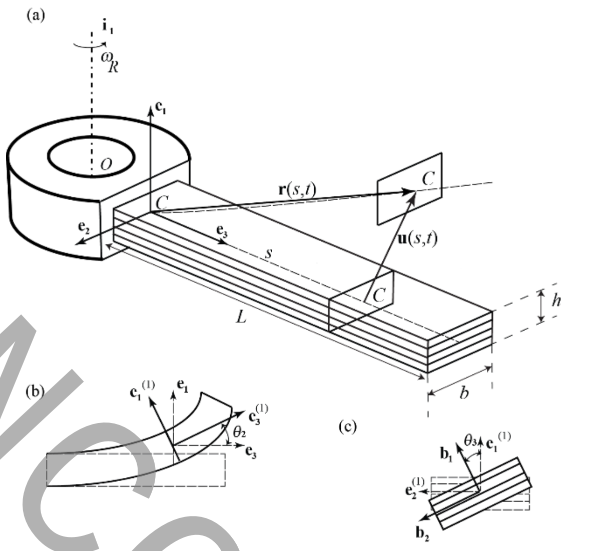

Fig. 1. (a)-Schematic of rotating composite beam, (b)-Interface coordinate system, (c)-Current coordinate system

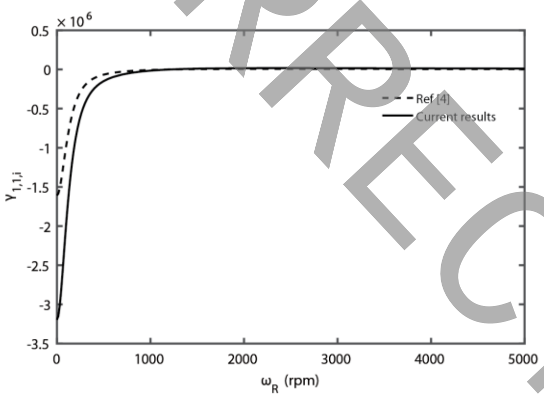

Fig. 2. Variations of the effective nonlinearity coefficient of the first flapping mode $\gamma_{1,1,i}$ [current results (solid-lines) and reference [4] results (dashed-lines)]

associated ENC. Thereafter, the real-valued amplitude and phase modulation equations are, respectively, read as:

$$a_k'(T_2) \simeq 0, \quad a_k(T_2)\beta_k'(T_2) = \frac{1}{4}a_k(T_2)^3\gamma_{1,k,i} \tag{1}$$

In which T_2 is the slowest time scale [3], and $\gamma_{1,k,i}$ is the k th ENC. Accordingly, the steady state solution leads to:

$$a_k(T_2) = Constant = a_k^0, \quad \beta_k(T_2) = \frac{1}{4}a_k^{0\,2}\gamma_{1,k,i}T_2 + \beta_k^0 \tag{2}$$

where a_k^0 and β_k^0 are defined using the initial conditions. Thereafter, the k th nonlinear natural frequency is achieved by $\omega_{1,k}^{NL} = \omega_{1,k} + 1/4\,a_k^{0\,2}\gamma_{1,k,i}$ in which $\omega_{1,k}$ is the k th linear natural frequency. It should be mentioned that the sign of $\gamma_{1,k,i}$ determines the softening and hardening treatment of the k th nonlinear natural frequency.

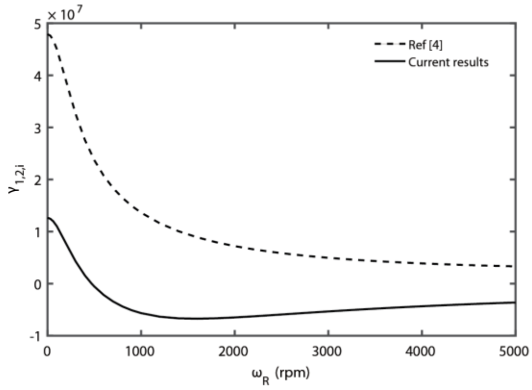

Fig. 3. Variations of the effective nonlinearity coefficient of the second flapping mode $\gamma_{1,2,i}$ [current results (solid-lines) and reference [4] results (dashed-lines)]

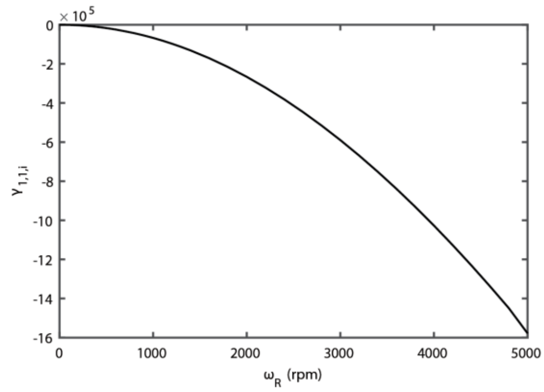

Fig. 4. Variations of the effective nonlinearity coefficient of the first torsional mode $\gamma_{1,1,i}$

4- Numerical Results

A rotating symmetric cross-ply laminated beam is considered for evaluation with layup $[0^\circ/90^\circ/90^\circ/0^\circ]$, $d_3 = 0.2(\text{m})$, $L = 2(\text{m})$, $h = 0.005(\text{m})$ and $b = 0.05(\text{m})$. The Young and shear moduli, and the Poisson's ratio and the mass density are, respectively, $E_1 = E_2 = 9.6(\text{GPa})$, $E_3 = 145(\text{GPa})$, $G_{12} = 3.4(\text{GPa})$, $G_{13} = G_{23} = 4.1(\text{GPa})$, $\nu_{31} = \nu_{32} = 0.3$, $\nu_{21} = 0.5$, $\rho = 1389\left(\text{kg/m}^3\right)$. The ENC of the first flapping mode $\gamma_{1,1,i}$ is shown in Fig. 2 in comparison with the results of reference [4]. It is worth to note that reference [4] has ignored the torsional motion by dropping the Coriolis force influence. The computed values for the ENC are different however the same sign is estimated. The ENC of the second flapping mode is depicted in Fig. 3. It is clear that after slow rotation speeds the continuous hardening treatment predicted by reference [4] alters to a softening behaviour in the present analysis.

The ENC of the first torsional mode is presented in Fig. 4. Fig. 4 illustrates that the ENC for stationary beams is zero however by increasing the rotating speed due to the coupling made by Coriolis force, it induces a monotonically enlarging softening treatment.

5- Conclusions

In this paper, the nonlinear free vibration of symmetric rotating composite un-shear able beams was studied regarding the flapping-torsional motions. The most important results read as:

1. The first torsional mode represents a softening treatment for rotating beams;
2. In the first flapping mode, the sign of the effective nonlinearity coefficient estimated in both cases, the flapping-torsional and the flapping-axial examinations, is the same.
3. The sign of the effective nonlinearity coefficient for the second flapping mode is identical for the flapping-torsional and the flapping-axial investigations just at low rotational speeds.

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