



Buckling Analysis of Truncated Conical Sandwich Panel with Porous Functionally Graded Core in Different Thermal Conditions

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ABSTRACT: In this paper, for the first time, by considering the flexibility of the core in the high order theory of sandwich shells, the buckling behavior of the truncated conical sandwich shells which include a temperature-dependent porous functionally graded core and two temperature-dependent homogeneous face sheets are investigated in various thermal conditions. The power-law rule which modified by considering the two types of porosity volume fractions is applied to model the gradual variation of functionally graded materials. By applying the principle of minimum potential energy, considering the in-plane stresses in the core and faces, and nonlinear von-Karman strains for both mechanical and thermal stresses, the governing equations are obtained under the axial in-plane compressive loads. A Galerkin procedure is used to solve the equations in a simply supported, clamped and clamped-free boundary conditions. Uniform, linear and nonlinear temperature distributions are used to model the effect of the temperature changing in the sandwich shell. To verify the results of these work, they are compared with finite element method results obtained by ABAQUS software and for special cases with the results in literature. Critical load variations are surveyed versus the temperature changing, geometrical effects, porosities, and some others in the numerical examples.

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1- Introduction

After the first considering sandwich constructions in a research paper in 1944, these modern structures with high flexural stiffness to weight ratio have become favorite structures among the researchers. Sandwiches include two faces to resist the in-plane and bending loads and a core to resist the transverse shear loads and maintain the faces distance [1]. Conical sandwich shells are important kinds of sandwich structures which have been applied in modern equipment such as vessels, missiles, spacecraft, and nuclear reactors. Despite the importance of these modern structures, due to the complex set of partial differential equations, there are little studies about conical sandwich shells comparing with cylindrical shells and circular plates [2].

Due to failure, delamination and thermal stress concentration in high-temperature environments, application of usual materials and ordinary composites is not proper. Japanese researchers proposed Functionally Graded Materials (FGM) to overcome these problems. FGMs are microscopic inhomogeneous materials which gradually graded from a metal surface to a ceramic one [3]. During the production process of FGMs, some micro-voids appear which affect the material properties. Considering these porosities in the modeling of these materials is a development in the researches. Considering the dependence of material properties on the temperature, and distribution of the temperature in the thickness direction of the structure are important to model the mechanical behavior of the sandwich panels, too.

There are several approaches to investigate the mechanical behavior of sandwich structures, such as the Finite Element Model (FEM), shear deformation theories, 3-Dimensional (3D) elastic theory, and energy methods [4]. In these theories the core height is constant, but in fact the thickness of the sandwich plates is variable. So, the core should be considered as a flexible layer that compressed transversely. In the classical theories, the localized effects in the core cannot be calculated, so to consider these effects, a high order theory was presented [5].

Often, studying the buckling of conical shells has been interesting for researchers. Sofiyev [6] by using First-Order Shear Deformable Theory (FSDT) and Galerkin method studied the stability of Functionally Graded (FG) conical sandwich shells under axial load. Xu et al. [7] studied the buckling behavior of truncated conical shells subjected to uniform pressure by using the calculus of variations. Zhong and Reimerdes [8] employed a high order theory to study the stability behavior of conical sandwich shells by using the principle of minimum total potential energy. Sofiyev [9] in a review paper gathered some research on vibration and buckling of FG shells.

2- Methodology

In this study, by applying a high order theory which modified by considering the flexibility of the core, buckling behavior of truncated conical sandwich shells with porous FG core under axial compressible loads is investigated in the uniform, linear and nonlinear temperature distributions. FG material properties are temperature and location-dependent which graded in according to power-law rules that include the volume fraction

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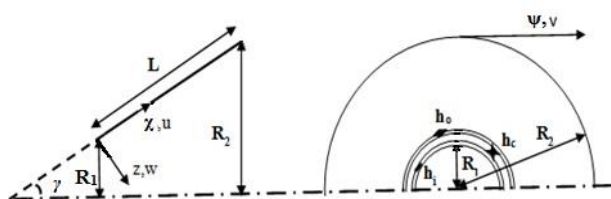


Fig. 1. Hybrid turbine model

of the porosities. The homogeneous materials are temperature dependent, too. High order stresses and thermal stress resultants, in-plane stresses and thermal stresses of the core and face sheets are considered at the same time. Nonlinear von Karman strains are used for both mechanical and thermal stresses. Boundary conditions are simply supported, clamped and clamped-free and equations are derived based on the minimum potential energy principle. To obtain critical loads, a Galerkin method is applied. To validate the present approach, the results of this analytical approach are compared with the numerical results obtained by ABAQUS software and for a special case are compared with some studies. Finally, the effects of the temperature changing, volume fraction distribution of FG core, porosity and some geometrical effects on the buckling characteristics of defined sandwich shells are investigated.

3- Results and Discussion

In this study, a truncated sandwich shell is considered which the interior plane of the core and inner face sheet are made of the zirconium dioxide and the outer plane of the core and outer face sheet are made of silicon nitride. Fig. 1 shows a schematic of a truncated conical sandwich shell in a curve linear coordination.

To study the buckling behavior of FG conical sandwich shells, and to obtain the governing equations, minimum potential energy principle is applied which includes total potential energy variation, δV , and internal potential energy variation, δU . Based on a high order sandwich shell theory, distinct displacement fields should be considered for each layer. So FSDT is employed to model the displacement fields of the face-sheets. Also, cubic patterns are applied to model the kinematic relations of the core with twelve unknown coefficients for the in-plane and vertical displacement components. By substituting the kinematic relations and compatibility conditions, and after some algebraic operations, the twenty-eight equations are obtained. A Galerkin method is determined to solve the governing equations of truncated conical sandwich shells, with twenty-eight trigonometric shape functions, which satisfy the boundary conditions. These twenty-eight equations are not independent and the number of them can be reduced by a reduction approach. Lagrange constants can be isolated as the expression of the face sheets. It can be seen that based on the compatibility conditions, the unknown constants of the faces are dependent on the core constants. At last by some operations, the number of the equations is reduced to sixteen in terms of the core unknown constants. These sixteen equations can be written in a 16×16 matrix. To verify the approach of this study, present

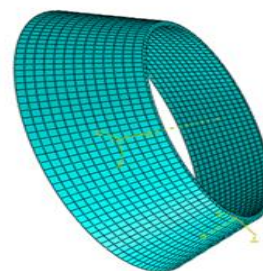


Fig. 2. The flow field and boundary conditions

results are compared with FEM results of ABAQUS software and in a special case are compared with results of Refs. [10-11]. Fig. 2 shows a finite element model of the structure in the present study.

4- Conclusions

Effects of temperature, power-law index, semi-vertex angle, thicknesses, radius, length, porosities and three temperature distributions on the critical load are discussed. Increasing the temperature reduces mechanical properties. So, with increasing the temperature in a constant power-law index, the critical load parameter decreases. While power law index is increased, the amount of ceramic reduces, so the critical load parameter decreases. In a constant power-law index and a constant temperature, the critical load parameter decreases when length to thickness ratio is increased. With increasing the semi vertex angle, the critical load parameter decreases. In a constant power-law index, the critical load parameter decreases when radius to thickness ratio is increased. With increasing the core to face-sheet thickness ratio in different power-law indices, the critical load parameters decrease, except in the zero power in which with increasing the ratio the critical load increases. In different power-law indices, with increasing the length to radius ratio, lower than four, the critical load parameters decrease. When the length to radius ratio is more than four, by increasing the ratio, the critical load parameters increase. With increasing the porosity volume fraction, the critical load parameter decreases. These decreasing are more in the case of even porosity distribution. Critical loads in the case of non-uniform distributions of temperature are bigger than the uniform one. The highest critical loads are in nonlinear temperature distribution. The critical load parameter is the highest in the clamped boundary condition.

References

- [1] J. Vinson, *The behavior of sandwich structures of isotropic and composite materials*, Routledge, 2018.
- [2] G. Jin, T. Ye, Z. Su, *Conical Shells*, in: *Structural Vibration*, Springer, 2015, pp. 199-233.
- [3] R.M. Mahamood, E.T. Akinlabi, *Functionally graded materials*, Springer, 2017.
- [4] J. Reddy, *Analysis of functionally graded plates*, *International Journal for numerical methods in engineering*, 47(1-3) (2000) 663-684.
- [5] Y. Frostig, M. Baruch, O. Vilnay, I. Sheinman, *High-order*

- theory for sandwich-beam behavior with transversely flexible core, *Journal of Engineering Mechanics*, 118(5) (1992) 1026-1043.
- [6] A. Sofiyev, The stability analysis of shear deformable FGM sandwich conical shells under the axial load, *Composite Structures*, 176 (2017) 803-811.
- [7] X. Jia-chu, W. Cheng, L. Ren-Huai, Nonlinear stability of truncated shallow conical sandwich shell with variable thickness, *Applied Mathematics and Mechanics*, 21(9) (2000) 977-986.
- [8] C. Zhong, H.-G. Reimerdes, Stability behavior of cylindrical and conical sandwich shells with flexible core, *Journal of Sandwich Structures & Materials*, 9(2) (2007) 143-166.
- [9] A. Sofiyev, Review of research on the vibration and buckling of the FGM conical shells, *Composite Structures*, 211 (2019) 301-317.
- [10] A. Sofiyev, The buckling of FGM truncated conical shells subjected to axial compressive load and resting on Winkler–Pasternak foundations, *International Journal of Pressure Vessels and Piping*, 87(12) (2010) 753-761.
- [11] P. Seide, Discussion: "Buckling of Circular Cones under Axial Compression" (Lackman, Leslie, and Penzien, Joseph, 1960, *ASME J. Appl. Mech.*, 27, pp. 458-460), *Journal of Applied Mechanics*, 28 (1961) 315.