



## Mechanical Behavior of Temperature-Sensitive Hydrogel Considering Functionally Graded Characteristics

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**ABSTRACT:** Hydrogels are 3 dimensional polymeric networks containing cross-linked chains which respond severely to the exterior stimuli and absorb a great amount of solution and swell. The functionally graded temperature-sensitive hydrogel is one of the most applicable materials to be used in the industry. Thus, to study the mechanical behavior of these materials, an energy density function is introduced which includes network stretch energy and mixing part. Considering the properties variation along the thickness direction, bending of functionally graded temperature-sensitive hydrogels is solved analytically under plane strain assumption. Verifying the presented analytical procedure, the results of this approach is compared with the outcomes of finite element method. To solve diverse problems by finite element method, UHYPER subroutine has been verified in the free-swelling problem. Next, the radius and stresses are studied by both methods for functionally graded temperature-sensitive hydrogels. Finally, according to the importance of factors such as semi-angle and bending curvature in industrial designs, these factors are investigated by changing the temperature in a range of 320 to 288 Kelvin. The continuity of the radial and tangential stresses field is the other reason for utilizing functionally graded hydrogels, while the multi-layer hydrogels do not have continuous stress fields.

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### 1. Introduction

Smart hydrogel is one of the most applicable materials which has drawn researchers' attention lately. This material contains 3Dimensional (3D) polymeric networks which react significantly to the environmental stimuli. In this regard, hydrogels absorb a plethora of solvent which led them to swell. This process is recoverable and makes these materials to be employed in diverse usages. The sensitivity of hydrogel to exterior stimuli are related to the composition of the hydrogels. The smart hydrogels can be exposed to various environmental stimuli including temperature, pH, light, glucose concentration and mechanical loadings [1-3]. According to the mentioned particular characteristic of smart hydrogels, they have been utilized in different application such as sensors and actuators, micro-fluidic switches, drug delivery, biomechanical applications and self-folding structures [4].

Temperature plays a crucial role in diverse industrial applications which cause the temperature-sensitive hydrogels becomes more important to the researchers. In this regard, investigation of the mechanical behavior of temperature-sensitive hydrogels was probed by considering various theories to define their responses under exterior temperature variation. Based on Flory-Huggins mixing energy, Chester and Anand [5] presented a constitutive model to describe the swelling of the temperature-sensitive hydrogels. After a while, Cai

and Suo [1] proposed a continuum-level constitutive model for temperature-sensitive Poly (N-IsoPropylAcrylAmide) (PNIPAAm) hydrogels and investigated the phase transition as well. Mazaheri et al. [6] pointed out the restriction of the constitutive model which was developed by Cai and Sue [1]. They modified this constitutive model to rectify the limitation of the model, including multiple solution and instabilities which occurs in the vicinity of the Phase Transition Temperature (PTT). They compared the obtained results not only with Cai and Sue [1] results and but also with experimental data which illustrated high accuracy adjacent to the PTT.

In this paper, considering Mazaheri et al. [6] constitutive model, the swelling behavior of temperature-sensitive hydrogel is investigated by considering characteristics variations. The mechanical behavior of the functionally graded hydrogel is studied when subjected to the temperature variation. Altering the temperature, the hydrogel, which is immersed in a solvent, swells dramatically. The swelling of the functionally graded hydrogel causes this structure to bend in a semi-annular shape. This problem is analyzed in analytic and finite element approaches.

### 2. Swelling of Temperature-Sensitive Hydrogel

In order to investigate the swelling behavior of a rectangular temperature-sensitive hydrogel of which the characteristics vary along the thickness, the following

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constitutive model is employed. In this model, the free energy density ( $W$ ) is defined to investigate the hydrogel responses to the temperature variation.

$$W = \frac{1}{2}G \left[ I - 3 - 2 \text{Log } J \right] + \frac{k_B T}{\nu} \left[ J - 1 \left[ -\frac{1}{J} - \frac{1}{2J^2} - \frac{1}{3J^3} + \frac{\chi J, T}{J} \right] \right] \quad (1)$$

where  $G = Nk_B T$  and  $\nu$ ,  $K_B$ ,  $T$  and  $N$  denote the solvent volume, Boltzmann's constant, temperature and chain density of the hydrogel. Considering  $X_0 = A_0 + B_0 T$  and  $X_1 = A_1 + B_1 T$ , the chain density is presented as:

$$\chi J, T = X_0 + \frac{X_1}{J} \quad (2)$$

where  $J$  is the determinant of the deformation gradient and  $A_0, A_1, B_0$  and  $B_1$  are material parameters.

Assuming a rectangular functionally graded hydrogel in Cartesian coordinate as a reference state and the deformed semi-annular structure as a current state, the deformation gradient tensor is defined as:

$$F_{1r} = \frac{dr}{dX_2}, F_{2\theta} = r \frac{2\bar{\theta}}{L}, F_{3z} = 1 \quad (3)$$

In which  $r(X_2)$  is defined as the current radius and  $\bar{\theta}$  is the semi-angle.

Considering the energy density function of the hydrogel, the radial and tangential stresses are calculated as:

$$\frac{P_i \nu}{k_B T} = N \nu \left[ \lambda_i - \frac{1}{\lambda_i} \right] + \left( \frac{-1/2 + X_0 - X_1}{\lambda_i J} + \frac{-1/3 + 2X_1}{\lambda_i J^2} - \frac{1}{\lambda_i J^3} \right) \quad (4)$$

Substituting the stress components in the equilibrium equation, a nonlinear equation is recast. Thus, utilizing the proper boundary conditions, free-stress state at inner and outer surfaces, the stress distribution of analytical solution is determined.

### 3. Results and Discussion

The problem of swelling of the functionally graded hydrogel was also implemented in finite element software ABAQUS. The material parameters utilized both in analytical and finite element methods are presented in Table 1.

Parameter	Value	Parameter	Value
$A_0$	-12.947	$A_1$	17.92
$B_0$	0.04496	$B_1$	-0.056

The radial and tangential components of stress are presented in both analytical and finite element methods. Comparing the results of methods in Figs. 1 and 2, the accuracy of the proposed analytical solution is verified.

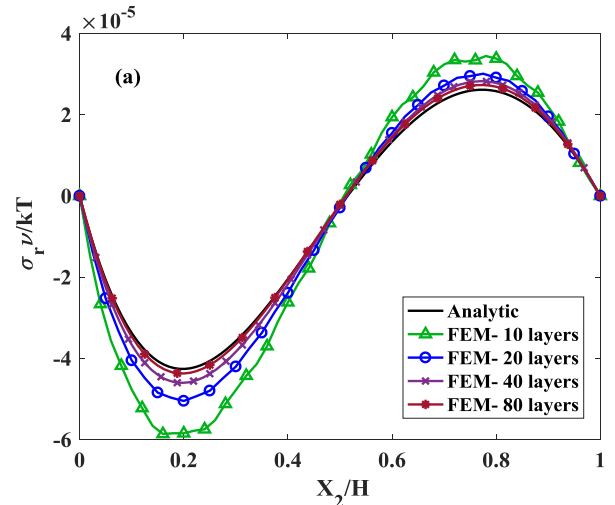


Fig. 1. Radial stress distribution of hydrogel

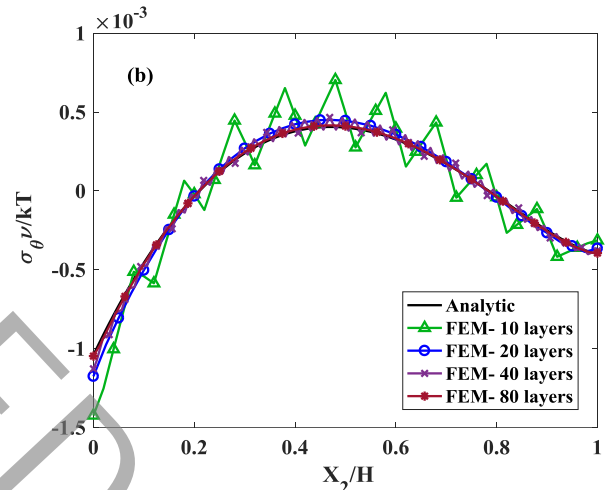
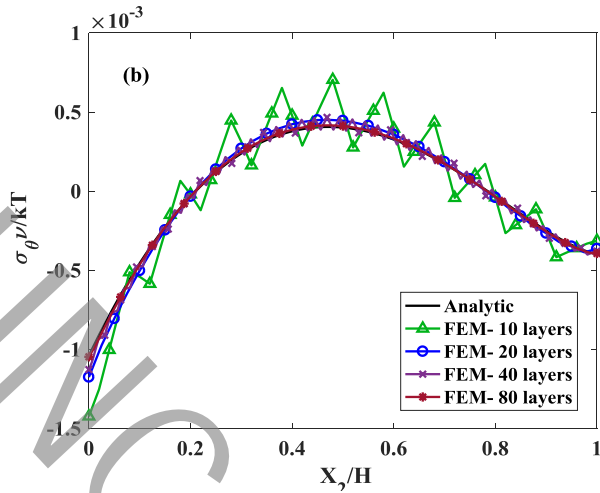


Fig. 2. Tangential stress distribution of hydrogel

For further validation, the deformation of the swollen hydrogel structure was also inspected. As depicted in Fig. 3, the computed current radius of the analytic solution is perfectly conformed to finite element results.

### 4. Conclusions

In this study, the swelling behavior of a rectangular temperature-sensitive poly(N-isopropylacrylamide) hydrogels is investigated by considering the cross-linked density variation along the thickness. Considering the free energy density and the equilibrium equation, a nonlinear equation is obtained which can be solved by assuming the free-stress state on the inner and outer surfaces of the semi-annular structure. The stress and strain analysis were performed utilizing both analytic and finite element approaches. The presented results illustrate the robustness and accuracy of the proposed solution. Increasing the number of strips in finite



**Fig. 3. Tangential stress distribution of hydrogel**

element analysis cause the results of the finite element to converge to the analytical ones. It is also apparent that the radial stress vanishes at the inner and outer surfaces which satisfies the boundary condition in both analytical and finite element methods.

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