



Transient Response of Annular Sandwich Plate with Functional Graded Core Combined with Piezoelectric Layers

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ABSTRACT: In this study, the transient response of the symmetric annular sandwich plate, with functionally graded core and piezoelectric layers, is investigated. It is also assumed that the sandwich plate is under external harmonic force and electrical voltage. Based on the power function model, it is assumed that the properties of the core material vary in the direction of the core thickness. To express the displacement field, the third order shear deformation theory is used. By use of the Hamilton principle, the structural equations are obtained in terms of displacement components and solved using the differential quadrature method. Finally, the time response is evaluated in terms of variations in effective parameters such as internal radius, power function index, core thickness and external voltage. The simulation results showed that the amplitude of the oscillations decreases when the internal radius of plate to be increased, in the desired time interval. In addition, by increasing the index parameter of the power function, the time response range increases. Finally, by applying external electrical voltage, the vibration amplitude of plate reduced and this advantage is used in control of vibrating systems.

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1- Introduction

Circular and annular composite plates are widely used in mechanical, construction, nuclear, submarine, aerospace and computer industries. In addition, due to the increasing use of piezoelectric materials in intelligent structures, many researchers focus on plates that have piezoelectric layers. viliani et al. [1] investigated the problem of buckling control of rectangular Functional Graded Material (FGM) plates with sensor and actuator layers. phung et al. [2] presented an efficient method for dynamic control of piezoelectric composite plates. In this study, the governing equations are obtained by the general Lagrangian method using the Van Denmark strains and solved by the Newmark numerical method. Narayanan and Balamurugan [3], using finite element method and first-order theory to controlled the vibration of plates and shells integrated with piezoelectric layers. Wang [4], investigated the symmetric bending of angular plates under uniformly distributed loads using classical theory and first-order shear deformation theory. Sahraee and Saidi [5] also used the third-order shear theory to study the symmetrical bending of circular FGM plates and to express the corresponding solutions in terms of classical theory. In the present paper, due to changes in the properties of FGM materials in thickness direction and the inherent thickness of the plate, a third order shear theory that is more accurate in such problems is used for modeling circular plate made of functional gradient material integrated with the Piezoelectric

layers under transient pressure and electrical loading.

2- Governing Equations

The geometry of the problem is shown in Fig. 1. The property of the functional materials is that the properties of the materials used in the structure are variable in thickness direction. Changes in the mechanical properties of the core are expressed by power law. On the other hand, the external load $q(r, t)$ is also logged in.

As stated above, the core layer is assumed to be a functionally graded material which is combination of ceramics and metal, so that it is pure ceramic at above of the core, ($z = \frac{h_c}{2}$), and pure metal at the bottom of the core ($z = -\frac{h_c}{2}$), (In this case, it is assumed that the properties of core material, such as the modulus of elasticity, (E), and mass density, $\rho(z)$, vary by thickness.

$$\begin{aligned} E(z) &= E_m + (E_c - E_m) \left(\frac{1}{2} + \frac{z}{h_e} \right)^k \\ \rho(z) &= \rho_m + (\rho_c - \rho_m) \left(\frac{1}{2} + \frac{z}{h_e} \right)^k \end{aligned} \quad (1)$$

According to the power function model, variation of core mechanical properties can be expressed as [6].

In these equations, the subtitle m and c respectively represent the metal (herein aluminum) and the ceramic (herein are silicon). K is an index of power function that has values greater than zero. The displacement field in r and z direction

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Table 1. Turbine characteristics

$\varphi = \frac{R_1}{R_2}$	Method	0.1	0.3	0.5
λ_1	Present Differential Quadrature Method (DQM)	19.65261	30.79739	52.51617
	Ritz (reference [9])	19.84	30.04	48.31
λ_2	Present DQM	46.88275	69.89355	113.9494
	Ritz (reference [9])	44.91	64.23	97.39

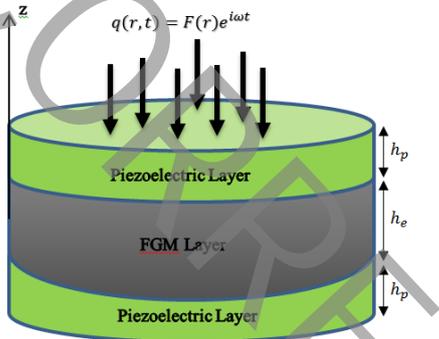


Fig. 1. Comparison of total moment vs. azimuth angle for 30 RPM

in terms of the Reddy theory (third-order shear deformation theory) for an arbitrary point of the structure is expressed as follows [7]:

$$\begin{aligned}
 u_r(r, z, t) &= u_0(r, t) + z\theta_r(r, t) - \frac{4z^3}{3h^2} \left(\theta_r(r, t) + \frac{\partial w_0(r, t)}{\partial r} \right), \quad h = \frac{h_e}{2} + h_p \\
 u_c(r, z, t) &= 0 \\
 u_z(r, z, t) &= w_0(r, t)
 \end{aligned}
 \tag{2}$$

Hamilton's principle is used to obtain the governing equations. Finally, the governing equations of motion are solved by the numerical solution of square differences, which was first proposed by Bellman and Casti [8] in 1971.

3- Results and Discussion

Prior to expressing the results, we will be able to verify the relations obtained in this paper. For this purpose, the first and second non-dimensional frequencies of a annular plate with clamped boundary conditions are obtained and compared with reference [9] in Table 1, for different radial ratios. Note that the first-order shear deformation theory and Rayleigh's numerical solution is used.

According to the results of Table 1, the accuracy of the obtained relationships can be observed. In the following, the time response when making changes to effective parameters

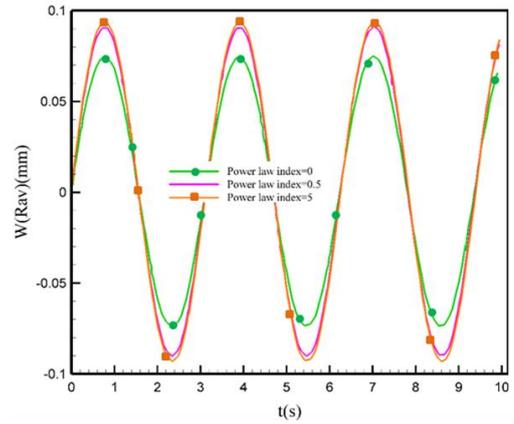


Fig. 2. The flow field and boundary conditions

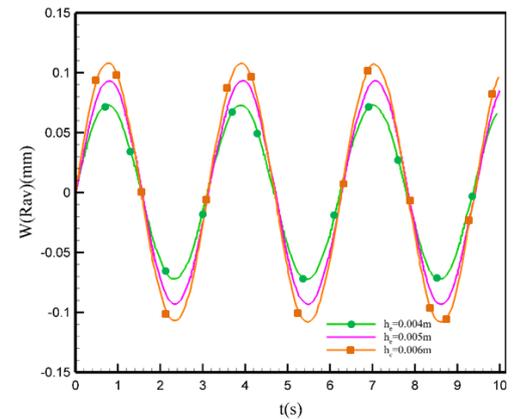


Fig. 3. Verification of straight-bladed turbine total moment coefficient

such as aspect ratio, power function index and applied voltage are discussed on the plate response. In Fig. 2, the effects of the power function index on the system response are shown. The power function index states that the properties of functional materials used in the core in terms of thickness differ and are a function of the properties of aluminum and silicon. As can be seen from the figure, with the increase of the index parameter, the amplitude of the time response increases.

In Fig. 3, the effects of the core thickness on the plate time response are investigated. As seen from this figure, the time response amplitude is increased by increasing the thickness of the core. It seems that, with increasing core thickness, the natural frequency of the sheet is close to the excitation frequency. Therefore, despite the increased hardness of the plate, the range of oscillations has increased.

4- Conclusion

Based on the results of this research, the following quantitative and descriptive results are obtained:

With increasing internal radius, the range of plate oscillations decreases in the desired time domain. This is due to an increase in the relative hardness of the plate, despite the

boundary conditions on the inner edge.

Increasing the index parameter of the power function increases the response time of the plate. This is due to the change in the properties of the core from ceramic to metal. In addition, by increasing the index, the power function responds to the plate response to an asymptote that represents the entire metal core.

Changing the thickness of the core increases the hardness of the plate, which naturally leads to a decrease in the amplitude of the response. In addition, due to the change in the plate frequencies and the harmonic excitation, in some thickness ratios, the amplitude has increased.

References

- [1] N. Viliani, S. Khalili, H. Porrostami, Buckling analysis of FG plate with smart sensor/actuator, (2009).
- [2] P. Phung-Van, L.B. Nguyen, L.V. Tran, T.D. Dinh, C.H. Thai, S. Bordas, M. Abdel-Wahab, H. Nguyen-Xuan, An efficient computational approach for control of nonlinear transient responses of smart piezoelectric composite plates, *International Journal of Non-Linear Mechanics*, 76 (2015) 190-202.
- [3] S. Narayanan, V. Balamurugan, Finite element modelling of piezolaminated smart structures for active vibration control with distributed sensors and actuators, *Journal of sound and vibration*, 262(3) (2003) 529-562.
- [4] X. Wang, Differential quadrature for buckling analysis of laminated plates, *Computers & structures*, 57(4) (1995) 715-719.
- [5] S. Sahraee, A. Saidi, Axisymmetric bending analysis of thick functionally graded circular plates using fourth-order shear deformation theory, *European Journal of Mechanics-A/Solids*, 28(5) (2009) 974-984.
- [6] C. Hong, Transient responses of magnetostrictive plates by using the GDQ method, *European Journal of Mechanics-A/Solids*, 29(6) (2010) 1015-1021.
- [7] H. Mozafari, A. Ayob, Effect of thickness variation on the mechanical buckling load in plates made of functionally graded materials, *Procedia Technology*, 1 (2012) 496-504.
- [8] R. Bellman, J. Casti, Differential quadrature and long-term integration, *Journal of Mathematical Analysis and Applications*, 34(2) (1971) 235-238.
- [9] K.-M. Liew, Y. Xiang, S. Kitipornchai, C. Wang, *Vibration of Mindlin plates: programming the p-version Ritz method*, Elsevier, 1998.