

# Simulation of Two-Phase Flow and Heat Transfer in a Channel and around a Tube by Lattice-Boltzmann Method

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## ABSTRACT

Determination of multiphase flow dynamics and thermal behavior of two-phase flow in a channel are of importance. The small-scale surface tension effect and related simulation efficiency, precision, and stability, have caused mesoscopic Lattice Boltzmann method broadening application. In the current study, the thermal-hydraulic behavior of subcooled falling flow in a vertical channel and around a single horizontal tube is simulated by using Lee method and Phase-Filed model, and thermal Passive Scalar model. The modified curved boundary conditions and two different boundary conditions for side boundaries are investigated. The density ratio is 20 and other property's ratios of water are applied, and the outside diameter of tube is 28.9mm. The flow, temperature, and pressure fields are presented and detailed understanding about the movement of the three-phase contact line, circulating flow and local and average Nusselt number are determined. The film thickness, thermal boundary layer variation by the film thickness, Reynold number effect on Nusselt number and mass conservation are investigated as verification. The results have shown good consistency and high effectiveness in the simulation of multiphase gas-liquid flows in the presence of a circular obstacle, and for viscosity and thermal diffusivity ratios of water.

## KEYWORDS

Lattice Boltzmann, Falling Film, Horizontal Tube, Two-Phase Flow, Heat Transfer.

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## 1. Introduction

Falling Film Flow and heat transfer around a horizontal heated tube have significant application in several industries. There are several interdependent mechanisms that have not been recognized yet. Narvaez et al. [1] reviewed experimental studies and noted that the available empirical correlations are strongly dependent on operating conditions under which they had been developed.

Moreover, dynamic behavior of microscopic phenomena at the interface is the main problem of numerical methods. Also, deficiency in calculation of small-scale surface tension and gradients cause the formation of parasitic currents and instabilities. Mirjalali et al. [2] reviewed and evaluated the most common classic methods. They concluded that phase-filed and Volume of Fluid<sup>2</sup> are most reliable methods. Meanwhile, there is not any comprehensive theoretical solution for this problem. Rogres et al. [3] investigated the falling film on single tube and applied several significant simplifications. Their results are applicable for limited ranges of non-dimensional numbers.

Mirjalali et al. [4] have done another study and showed the superiority of phase-filed model in comparison to volume of fluid, in terms of accuracy and stability. They concluded that the biggest remaining challenge is development of a stable more cost-effective model. Therefore, it could be concluded that because of application of Phase-Field model in most of Lattice Boltzmann Methods<sup>3</sup>, and because of molecular kinetic nature of LBM, it is the most effective method for simulation of two-phase flows. In this regard several studies have compared LBM with classic numerical methods[5-7]. Their results have approved the precision, stability, time-efficiency, simplicity and applicability of LBM.

Due to kinetic instinct of LBM, instability increases with increase of differences in properties of phases. Lee [8] in 2009 introduced a model which is stable up to density ratio of 1000. In this paper, the thermal and flow behavior of a jet of water at 100°C that flows around a horizontal tube at 110°C under the gravity force is simulated by multiphase Lee's method and Passive Scalar Method. The modified curved boundary treatments are used. Finally, the flow and temperature fields are presented and investigated.

## 2. Methodology

In Lee's method, two distribution functions  $h$  and  $g$  are used to simulate flow behavior of two immiscible and incompressible phases. The final Discrete Boltzmann Equation for the evolution of the hydrodynamic pressure and momentum,  $g$ , is

$$\bar{g}_\alpha(\mathbf{x} + \mathbf{e}_\alpha \delta t, t + \delta t) - \bar{g}_\alpha(\mathbf{x}, t) = -\frac{1}{\tau + 0.5} (\bar{g}_\alpha - \bar{g}_\alpha^{eq}) \Big|_{(\mathbf{x}, t)} + \delta t (\mathbf{e}_\alpha - \mathbf{u}) \cdot [\nabla \rho c_s^2 (\Gamma_\alpha - \Gamma_\alpha(0)) + (-C \nabla \mu + F_{ext}) \Gamma_\alpha] \Big|_{(\mathbf{x}, t)} \quad (1)$$

where  $\bar{g}_\alpha$  and  $\bar{g}_\alpha^{eq}$  are particle and equilibrium distribution functions respectively:

$$\bar{g}_\alpha = g_\alpha + \frac{(g_\alpha - g_\alpha^{eq})}{2\tau} - \frac{\delta t}{2} (\mathbf{e}_\alpha - \mathbf{u}) \cdot [\nabla \rho c_s^2 (\Gamma_\alpha - \Gamma_\alpha(0)) + (-C \nabla \mu + F_{ext}) \Gamma_\alpha] \quad (2)$$

$$\bar{g}_\alpha^{eq} = g_\alpha^{eq} - \frac{\delta t}{2} (\mathbf{e}_\alpha - \mathbf{u}) \cdot [\nabla \rho c_s^2 (\Gamma_\alpha - \Gamma_\alpha(0)) + (-C \nabla \mu \Gamma_\alpha + F_{ext})] \quad (3)$$

In the same way, the Discrete Boltzmann Equation for the transport of composition ( $h$ ) can be written in the following form:

$$\bar{h}_\alpha(\mathbf{x} + \mathbf{e}_\alpha \delta t, t + \delta t) - \bar{h}_\alpha(\mathbf{x}, t) = -\frac{1}{\tau + 0.5} (\bar{h}_\alpha - \bar{h}_\alpha^{eq}) \Big|_{(\mathbf{x}, t)} + \delta t (\mathbf{e}_\alpha - \mathbf{u}) \cdot \left[ \nabla C - \frac{C}{\rho c_s^2} (\nabla p_h + C \nabla \mu - F_{ext}) \right] \Gamma_\alpha \Big|_{(\mathbf{x}, t)} \quad (4)$$

where the modified distribution functions  $\bar{h}_\alpha$  and  $\bar{h}_\alpha^{eq}$  are written as follows:

$$\bar{h}_\alpha = h_\alpha + \frac{(h_\alpha - h_\alpha^{eq})}{2\tau} - \frac{\delta t}{2} (\mathbf{e}_\alpha - \mathbf{u}) \cdot \left[ \nabla C - \frac{C}{\rho c_s^2} (\nabla p_h + C \nabla \mu - F_{ext}) \right] \Gamma_\alpha \quad (5)$$

$$\bar{h}_\alpha^{eq} = h_\alpha^{eq} - \frac{\delta t}{2} (\mathbf{e}_\alpha - \mathbf{u}) \cdot \left[ \nabla C - \frac{C}{\rho c_s^2} (\nabla p_h + C \nabla \mu - F_{ext}) \right] \Gamma_\alpha \quad (6)$$

where  $C = \rho_{local} / \rho$ , and  $h_\alpha = (C / \rho) f_\alpha$ . The Lattice Boltzmann Equation for the distribution function of temperature is:

$$\bar{s}_\alpha(\mathbf{x} + \mathbf{e}_\alpha \delta t, t + \delta t) - \bar{s}_\alpha(\mathbf{x}, t) = -\frac{1}{\tau_t + 0.5} (\bar{s}_\alpha - \bar{s}_\alpha^{eq}) \Big|_{\mathbf{x}, t} \quad (7)$$

Where Equilibrium distribution is presented as the following form

$$s_\alpha^{eq} = \omega_\alpha T \left( 1 + \frac{\mathbf{e}_\alpha \cdot \mathbf{u}}{c_s^2} \right) \quad (8)$$

When a boundary is located in the middle of the fluid node  $f$  and solid node  $b$ , the post-collision unknown distribution functions are determined based on the distance from  $f$  to the curved boundary ( $\Delta$ ) as below:

<sup>2</sup> VOF

<sup>3</sup> LBM

$$\Delta = \frac{|x_f - x_w|}{|x_f - x_b|} \quad 0 \leq \Delta \leq 1 \quad (9)$$

$$f_i(x_b + c_i \Delta t, t + \Delta t) = (1 - \chi) f_i(x_f + c_i \Delta t, t + \Delta t) + \chi f_i^*(x_b, t) \quad (10)$$

$$f_i^*(x_b, t) = f_i^{eq}(x_f, t) + \omega_i \rho(x_f, t) \frac{3}{c^2} e_i (u_{bf} - u_f) \quad (11)$$

$$u_{bf} = u(x_{ff}, t), \quad \chi = \frac{2\Delta - 1}{\tau - 2}, \quad \Delta < \frac{1}{2} \quad (12)$$

$$u_{bf} = \frac{1}{2\Delta} (2\Delta - 3) u_f + \frac{3}{2\Delta} u_w, \quad \chi = \frac{2\Delta - 1}{\tau + \frac{1}{2}}, \quad \Delta \geq \frac{1}{2} \quad (13)$$

Also, Post-collision temperature curved boundary conditions of Guo et al. [9] are employed.

### 3. Discussion and Results

The efficiency of method in calculation of the surface tension which plays major role in multiphase problems is verified by Laplace Law. The result shows less than 5% error at initial time steps.

Also, the verification of temperature-flow simulation is done by simulation of Rayleigh-Benard Convection. Fig. 1 shows expected mechanical and thermal dependencies, based on rotating flows and temperature profiles.

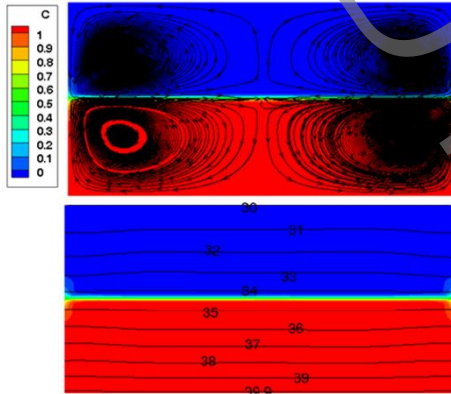


Fig. 1. Flow and Temperature Lines of Two-Phase Gas-Liquid Rayleigh-Benard Convection for  $Ra=2.9 \times 10^3$

Moreover, the effect of the side boundaries' conditions on flow and temperature behavior of falling liquid flow in a channel of gas are investigated. The order of falling flow velocity and the variation of pressure and temperature distribution are in line with expectations. Pressure changes in accordance with depth and the temperature of liquid increases as it contacts to heated wall.

The simulation of falling flow around the cylinder is done successfully and the details of flow behavior including the time evolution and location of circulating flow regarding to pressure distribution and location of three phase interface is determined. These results show the accuracy of curved boundary conditions.

Finally, by restricting the inlet size, the falling film is simulated. The time evolution of film entrance to the domain, formation of film around the tube, and separation of drops from the tube are presented in Fig.

2. The diameter of separated drop is validated by Yung equation [10].

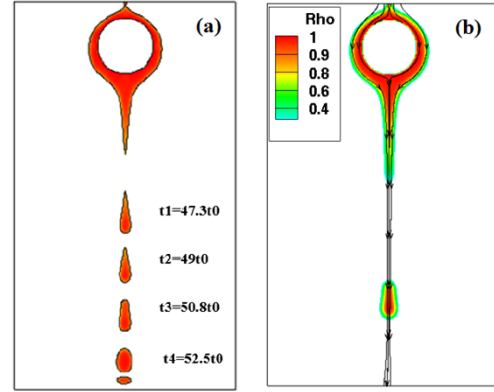


Fig. 2. (a) Time Evolution of Falling Film Formation, Separation and Leaving the Domain, (b) Falling Film Streamlines at  $t_3$

This simulation is done by considering Prandtl number of two phases and solving the temperature equation for both phases, without any simplification. As Fig. 3. Shows that the thermal boundary layer thickness increase with increment of circumferential angles in accordance with theoretical results [11, 12].

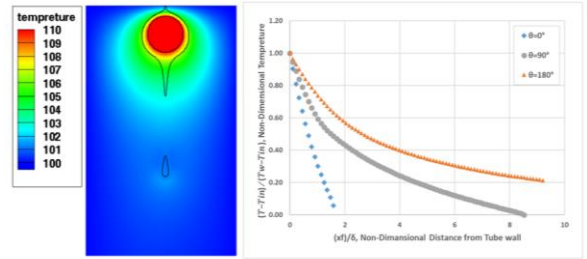


Fig. 3. Temperature Distribution around the Tube for Steady Film at different Circumferential Angles.

Local Nusselt Number is calculated by  $Nu = \frac{hD}{\kappa_f} = \frac{\partial T / \partial r|_{r=0} D}{T_w - T_m}$  and the average Nusselt number is equal  $\overline{Nu} = \frac{1}{\pi} \sum_0^\pi Nu = 2.18$ . This value is equal to 3.66 for

single phase flow in constant temperature tubes and 4.36 for constant heat flux. Also, based on Peclet and Prandtl Number of simulations, the comparison of results with similarity solution of natural convection is considerable. This value is  $\overline{Nu}_D = \left\{ 0.6 + \frac{0.387 Ra_D^{1/6}}{[1 + (0.559 / Pr)^{9/16}]^{4/27}} \right\}^2 = 4.1$  for flooded cylinder

in the pool of liquid [13]. The main reason of deviations could be the application limitations of correlations.

### 4. Conclusions

In this study, the flow and temperature behavior of two-phase falling flow around a horizontal tube has been simulated by the state-of-the-art Lattice Boltzmann Method. The performance of boundary conditions and stability of models for determined geometrical and

physical parameters have been concluded. The verifications including Laplace Law, Rayleigh-Benard Convection, and falling flow in vertical channel have showed the accuracy of method. By presenting the flow and temperature field of falling film, the formation and separation of film, and the local and average heat transfers are captured correctly. In Conclusion, despite limitation of stability due to property ratios and gradients, this method present reliable results.

## 5. References

- [1] B. Narváez-Romo, J.R. Simões-Moreira, Falling Film Evaporation: An Overview, in: Proceedings of 22nd International Congress of Mechanical Engineering–COBEM, 2013, pp. 3-7.
- [2] S. Mirjalili, S.S. Jain, M. Dodd, Interface-capturing methods for two-phase flows: An overview and recent developments, Center for Turbulence Research Annual Research Briefs, (2017) 117-135.
- [3] J. Rogers, Laminar falling film flow and heat transfer characteristics on horizontal tubes, The Canadian Journal of Chemical Engineering, 59(2) (1981) 213-222.
- [4] S. Mirjalili, C.B. Ivey, A. Mani, Cost and accuracy comparison between the diffuse interface method and the geometric volume of fluid method for simulating two-phase flows, in: APS Meeting Abstracts, 2016.
- [5] L. Scarbolo, D. Molin, P. Perlekar, M. Sbragaglia, A. Soldati, F. Toschi, Unified framework for a side-by-side comparison of different multicomponent algorithms: Lattice Boltzmann vs. phase field model, Journal of Computational Physics, 234 (2013) 263-279.
- [6] S. Ryu, S. Ko, A comparative study of lattice Boltzmann and volume of fluid method for two-dimensional multiphase flows, Nuclear Engineering and Technology, 44(6) (2012) 623-638.
- [7] S. Mukherjee, A. Zarghami, C. Haringa, K. van As, S. Kenjereš, H.E. Van den Akker, Simulating liquid droplets: A quantitative assessment of lattice Boltzmann and Volume of Fluid methods, International Journal of Heat and Fluid Flow, 70 (2018) 59-78.
- [8] T. Lee, Effects of incompressibility on the elimination of parasitic currents in the lattice Boltzmann equation method for binary fluids, Computers & Mathematics with Applications, 58(5) (2009) 987-994.
- [9] Z. Guo, C. Zheng, B. Shi, An extrapolation method for boundary conditions in lattice Boltzmann method, Physics of fluids, 14(6) (2002) 2007-2010.
- [10] R. Mei, D. Yu, W. Shyy, L.-S. Luo, Force evaluation in the lattice Boltzmann method involving curved geometry, Physical Review E, 65(4) (2002) 041203.
- [11] Z.-H. Liu, Q.-Z. Zhu, Heat transfer in a subcooled water film falling across a horizontal heated tube, Chemical Engineering Communications, 192(10) (2005) 1334-1346.
- [12] A. Kumar, Numerical Study of Falling Film Thickness on Horizontal Circular Tube-A CFD Approach, Int J Adv Technol, 7(169) (2016) 2.
- [13] A. Bejan, Convection heat transfer, John wiley & sons, 2013.