

Analysis of dynamic instability in sandwich thick beams with flexible functional core subjected to a follower force

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ABSTRACT

Aerial structures under non-conservative forces especially follower loads, may be exposed to dynamic or static instabilities. Thus, it is essential to design these structures so that it would prevent this phenomenon. In this paper, for the first time, dynamic instability of a thick sandwich beam with flexible core under follower force is considered using high-order theory of sandwich beams. In the present paper, shear and normal core plate stresses are also considered, which have been ignored in higher-order sandwich panel theory and improved higher-order sandwich panel theory. The sandwich beam consists of two surfaces and a flexible core. The common surface of the core with the surfaces comprises a complete connection, capable of withstanding shear and vertical stresses. Sandwich beam is considered as a linear elastic structure with small rotations and deformations. Equations of Motion of high-order sandwich beams under follower force, are derived using Hamilton's principle. The Beam fluttering phenomenon is investigated by applying boundary conditions and using a generalized differential quadrature method. In addition to the verification of results, effects of the beam's geometry and mechanical parameters have been studied. These results revealed that the threshold flutter force of the sandwich beam is similar to Timoshenko one.

KEYWORDS

Pneumatic structures, sandwich structures, dynamic instability, numerical solution method, flexible functional core

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1. Introduction

Investigating the behavior and dynamic stability of structures under the influence of the follower force is a topic of interest for many researchers. For the first time, the problem of elastic structure stability under the influence of the follower force was studied by Nikolai [1]. Investigation of the follower forces in structures has become of serious interest for researchers when three classic problems were simultaneously stated by Beck, Leipzig, and Hager, which became well-known under their own names. So far, few studies have been performed on the dynamic stability of sandwich structures influenced by follower forces and the behavior of sandwich structures with a targeted flexible core under the traction forces has not yet been investigated. In this paper, for the first time and using the improved higher order sandwich panel theory (IHSAPT) [2], we investigate the dynamic instability and probability of fluttering phenomenon in thick sandwich cantilever beam under compressive follower force.

2. Methodology

The three layer higher order theory used in the present paper is a polynomial model based on the distribution of core movements in terms of thickness, which is based on the results of first Frostig's model, except that the polynomial coefficients are considered unknown. The mathematical formulation of the higher order sandwich panel theory for unidirectional panels and sheets is available in [4-6]. The sandwich beam is considered linear elastic with small displacements and consists of a core with two thin beams and bending rigidity. The core surface is in full connection with the thin beams capable of withstanding shear and vertical stresses. External load can also be applied to the upper or lower surfaces. Figure 1 shows the structure of the beam.

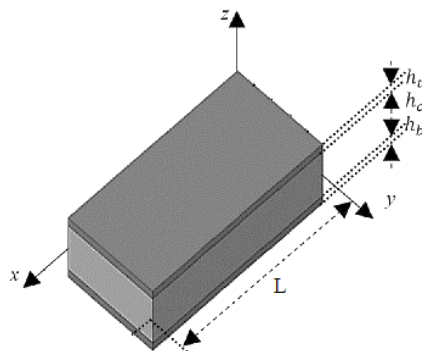


Figure 1. Geometrical properties of Beck's sandwich beam

Using the first order of the shear theory, the displacement field on the surfaces is considered as in Equations (1) [7].

$$u_j(x, z, t) = u_0^j(x, t) + z\psi_x^j(x, t) \quad (1- a)$$

$$w_j(x, z, t) = w_0^j(x, t), \quad j = t, b \quad (1- b)$$

For the beam core, the displacement field equations are in the form of Equations (2) [3].

$$u_c(x, z, t) = u_0(x, t) + z_c u_1(x, t) + z_c^2 u_2(x, t) + z_c^3 u_3(x, t) \quad (2- a)$$

$$w_c(x, z, t) = w_0(x, t) + z_c w_1(x, t) + z_c^2 w_2(x, t) \quad (2- b)$$

With the displacement functions defined in Equations (1) and (2), the strain and stress distributions are determined in each of the layers. The required dynamic equations, rotational inertia, and boundary conditions are derived from the Hamiltonian principle [3]. Boundary conditions of the Beck's beam are derived from the Hamiltonian principle; the values of the unknown displacement functions at the two boundary points of X_1 and X_2 are calculated from the value of the function at other points. After applying boundary conditions at the beginning and end points of the network, the number of unknowns decreases to $9(N-2)$. To solve the reduced equation, eigenvalues for different values of P force are calculated and, given the values of oscillatory frequencies, the stability or instability of the beam motion is determined under the influence of the follower force. Due to lack of access to an analytical solution for the existing differential equation system, the generalized differential quadrature¹ method is used [8]. In this method, the first and second order derivatives of each uniform and differentiable function of $f(x)$ at point $x = x_i$ are approximated by Equations (3) using Taylor expansion [8].

$$\frac{df(x_i)}{dx} = \sum_{j=1}^N a_{ij} f(x_j) \quad (3- a)$$

$$\frac{d^2 f(x_i)}{dx^2} = \sum_{j=1}^N b_{ij} f(x_j) \quad (3- b)$$

In Equations (3), N is the number of network grid points. As can be observed, in this method, the derivatives of the function are calculated at each point of the weighted sum of the function itself at network grid points. The length of the beam is disrupted by Equation (4).

$$X_i = a(1 - \cos[(2i - 3)\pi / 2(N - 2)]) / 2, \quad (4)$$

$$i = 2, 3, \dots, N - 1$$

¹ GDQ - Generalized Differential Quadrature Method

The set of equations of motion of the beam converts into the problem of eigenvalues of Equation (5).

$$[K]\{X\} = \omega^2 [M]\{X\} \quad (5)$$

where the square matrices of $[M]$ and $[K]$ represent the stiffness matrix and beam mass matrix, respectively. The unknown vector of $\{X\}$ also contains the values of the displacement functions at the network grid points. To check the validity and validation process, the results of the present study are compared with the results in [9].

3. Results and Discussion

At times when the imaginary part of the first and second frequencies is equal, the system is at the threshold of instability and the flutter phenomenon occurs. Variations of the frequencies of the first and second oscillations of a high order sandwich beam for two different values of the length to the core thickness ratio are shown in Figure 2. As observed, with the increment of the length to the core thickness ratio, the force and frequency of the flutter will increase. This increment in the force and frequency is due to change of beam becoming thinner and taller, in which with the increment of the value of the length to core thickness ratio, the beam will be considered taller and thinner and a large amount of the force is spent on swinging the beam.

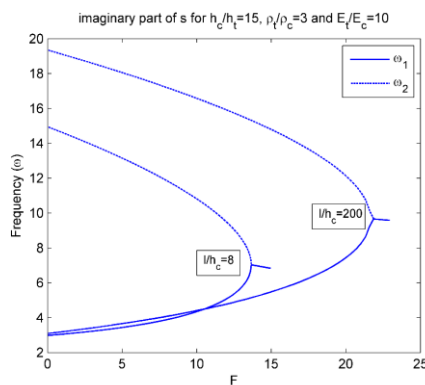


Figure 2. Variation of the frequencies of the first and second oscillations of different values of the length to core thickness ratio.

In Figure 3, effect of core thickness to shell thickness ratio on the instability threshold force value of a symmetric sandwich beam is investigated. Accordingly, in the long beams, as the shell thickness increases, a larger proportion of the follower force is attributed to their oscillation, resulting in a larger instability threshold force.

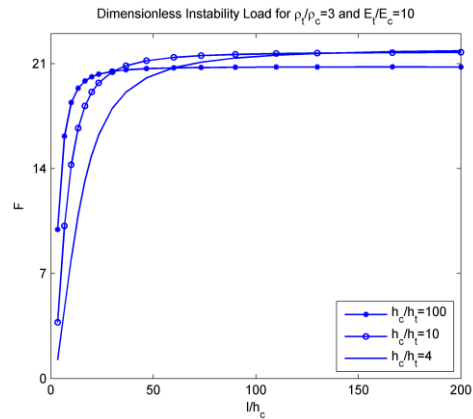


Figure 3. Variation of the dimensionless flutter force with the length to core thickness ratio for different core thickness to shell thickness ratios.

4. Conclusion

The results of this research are presented below:

- 1- In tall beams, the threshold value of the fluttering phenomenon tends to the corresponding results in Timoshenko's beam.
- 2- When the imaginary part of the first and second frequencies becomes the same, the system is on the verge of instability and the fluttering phenomenon occurs.
- 3- In tall beams, as the thickness of the surfaces increases, the amount of instability threshold force increases.
- 4- In sandwich beams, the force and the flutter frequency increase, as the length to core thickness ratio increases

5. References

- [1] E. Nikolai. On the stability of the rectilinear form of equilibrium of a bar in compression and torsion. *Izv Leningr Politech.* (1928).
- [2] K. Malekzadeh, M.R. Khalili, R.K. Mittal. Local and Global Damped Vibrations of Plates with a Viscoelastic Soft Flexible Core: An Improved High-order Approach. *Journal of Sandwich Structures and Materials*, 7 (2005) 431-456.
- [3] Y. Frostig, O. T. Thomsen. high-order free vibrations of sandwich panels with a Flexible core. *Journal of solid and structure*, 41(2004)1697-1724.
- [4] Y. Forsting, M. Baruch, O. Vinay, i. shteinman, i. higher-order theory for sandwich beams behavior with transversely flexible core. *journal of engineering mechanics*, 118 (1992)1026-1043.

[5] Y. Forsting, M. Baruch. free vibration of sandwich beams with a transverse flexible core: a higher order approach. *journal of solids and vibration*, 176 (1994)195-208.

[6] Frostig, Y.. Buckling of sandwich panels with a flexible core-high-order theory. *International Journal of Solids and Structures*. 35 (1998)183–204.

[7] Reddy JN (2003) *Mechanics of laminated composite plates and shells, theory and application*. CRC Press, Boca Raton.

[8] Shu, Chang. *Differential quadrature and its application in engineering*. Springer Science & Business Media, (2012).

[9] V. K. Goyal, R. K. Kapania. Dynamic stability of laminated beams subjected to non-conservative loading. *Thin-Walled Structures*, 46 (2008) 1359-1369.

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