

# Stability analysis of shear-thinning Flow in narrow gap Taylor–Couette axial flow

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## Abstract

In this study, by considering the fixed outer cylinder and the rotational and axial velocity of the inner cylinder in the Taylor–Couette flow, the analysis of shear thinning non-Newtonian fluid Carreau-Bird model motion is used to estimate flow parameters such as velocity and pressure distribution and predict dynamic fluid behavior and stability. The solution of the governing equations including continuity and momentum equations in the cylindrical system is used to obtain the velocity and pressure field. The base flow velocity field is obtained by solving the governing equations by assuming a narrow gap and applying the boundary conditions of the problem and the deviational flow velocity field after simplifying the nonlinear PDE system using the Galerkin Projection method. It comes with four unknowns. By solving the system of nonlinear differential equations in unstable conditions as well as determining the status of the root of the system characteristic equation, the dynamic behavior of the flow and its stability under different conditions of the Taylor number control parameter, non-Newtonian fluid and Reynolds axial predicted values. And to better understand the, the Lyapunov exponent method is also used for better analysis.

## Keywords:

Non Newtonian fluid, Taylor–Couette, Carreau-Bird model, Galerkin Projection method, Taylor Vortex Flow

## 1. Introduction

The viscous fluid rotational flow confined in the gap between two coaxial cylinders is known as the Taylor–Couette flow (TCF). This flow has become one of the important flows in experimental and theoretical hydrodynamics due to the rotation of one or both cylinders. It has a number of practical applications, such as in journal bearings, rotating machines, polymer processing, catalytic chemical reactors, extractors, filtration devices, oil and gas well drilling, etc.

In a groundbreaking study on Newtonian fluids by Taylor in 1923, it was shown that when the angular velocity of the inner cylinder increases above a certain threshold, the Couette flow becomes unstable. A secondary steady state, characterized by axisymmetric toroidal vortices, known as Taylor vortex flow (TVF), emerges after passing this point [1]. As the angular speed of the cylinder increases, the system undergoes a progression of instabilities that lead to states with greater spatio-temporal complexity, such as wavy vortex flow and spiral vortex flow [2,3]. Obviously, the TCF remains stable and purely azimuthal or circular Couette flow (CCF) at low angular velocity.

The problem of TCF of nonlinear fluids with axial flow has been discussed at this point. This study examines the TCF in conjunction with the axial flow induced by a translational movement of the inner cylinder, independent of its azimuthal flow. Of the very few studies found in literature, experiments or simulations have examined only the flow of Newtonian fluids between coaxial

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cylinders with axial flow. Li and Khayat [4] studied shear thinning fluid flow in the narrow gap between two cylinders with regard to the boundary conditions of rigid velocity for vortices in a research. In order to solve equations of conservation, they used the Galerkin method with respect to the Chandrasekhar functions. Ashrafi published a paper and discussed the sustainability of Shear Thinning fluid in the space between two rotating cylindrical. [5] Ashrafi and Hazbavi in 2011 presented the results of their research on stability of Shear Thinning fluid in Taylor-Couette flow geometry, considering the pressure gradient and still outer cylinder and discussed sustainability criteria. Also they investigated the turbulence in Non-Newtonian fluids in Taylor-Couette geometry and studied the initial conditions of stability of Non-Newtonian fluids using Lyapunov exponents. [6]

## **2-Methodology**

The total flow is defined as the sum of the base flow and deviation. The base flow velocity field is obtained by solving the governing equations by assuming a narrow gap and applying the boundary conditions of the problem and the deviated flow velocity field after simplifying the nonlinear partial differential equation system using the Galerkin Projection method and then solving the nonlinear differential equation system it comes with four unknowns.

The problem under study in this study is the Classic Taylor-Couette flow, to which the internal cylindrical motion has been applied. In this study, the effect of Non-Newtonian viscosity on rotational flow stability is investigated. The Carreau-Bird model will also be used to determine the viscosity dependence on strain rate. In the presence of axial internal cylindrical motion and the effect of non-Newtonian fluid index, it is investigated.

## **3-Result and Discussion**

To a certain Taylor number, this stream is a Couette flow type. Upon reaching the Taylor flow to a certain value called the Taylor Critical, we will see Taylor vortices flow (TVF). To a certain extent of the Taylor flow, velocity at different points in the flow is only a function of  $r$  and  $z$  (spherical coordinates) and is independent of time.

The radial velocity contours exhibit the same periodic behavior on both external and internal cylinders. Also, the maximum radial velocity is in the central part of the edges. As the walls of the inner and outer cylinders approach, the velocity decreases due to the formation of the boundary layer and the radial velocity on the walls is equal to zero. For the tangential velocity, it can be seen that the numerical value of velocity decreases with the distance from the inner cylinder to the outer cylinder. The placement of the maximum and minimum values of velocities in a state independent of time is independent of the value of Taylor number.

Two consecutive vortices are reversed relative to each other, that is, if one vortex is clockwise, the next vortex will be counterclockwise. The radial velocity component that does not exist in the base flow appears in TVF. In the axial velocity contours, the extremes are symmetric, and the values of the extremes are also symmetric. The high pressure area on the outer cylinder and the low pressure area change on the inner cylinder. The axial Reynolds increase in a fixed Taylor number permeates and

extends the extreme pressure region in the flow field from the outer cylinder to the inner cylinder. With the increase in the Taylor number, we will see that the nature of the flow regime changes and will no longer be independent of time.

In Figures 1 and 2, the velocity components for the Taylor-Couette flow for the time-independent mode are given in two states without and with the axial motion of the internal cylinder. Taylor's vortices are clearly recognizable in the figures presented.

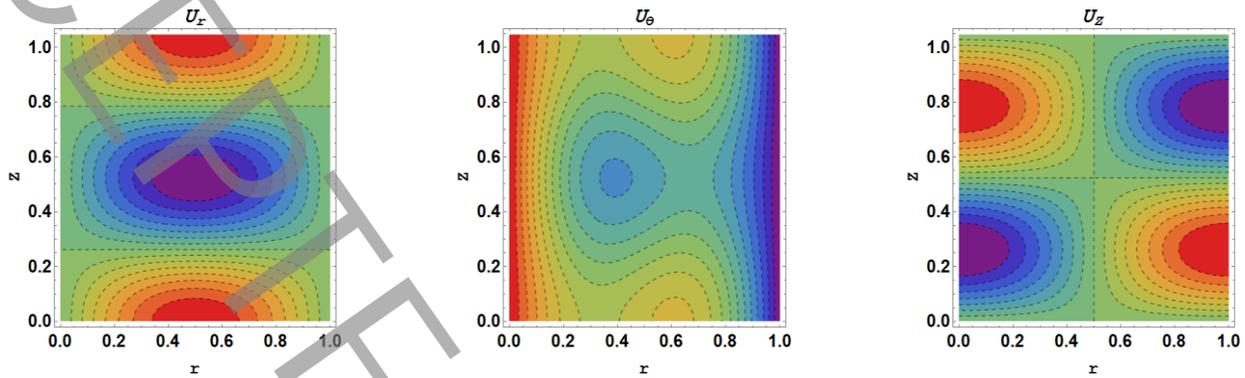


Fig.1 Velocity component contours without inner cylinder axial velocity

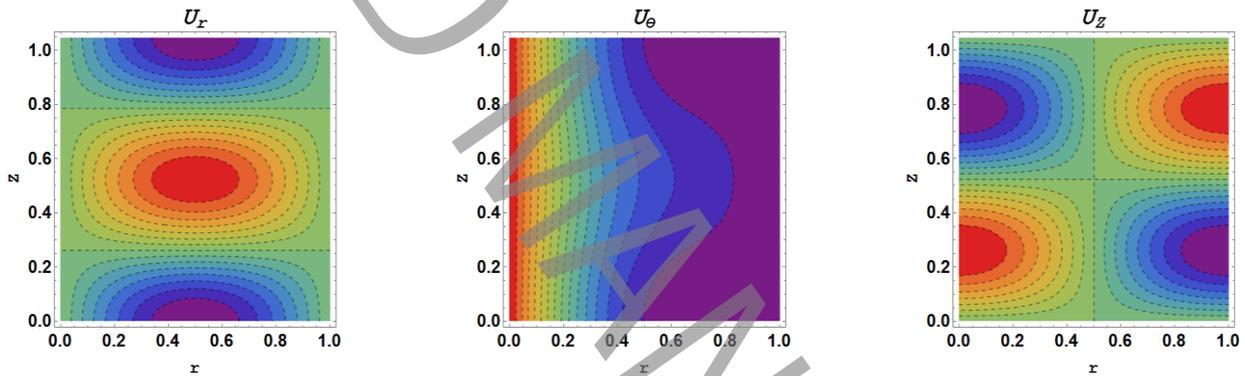


Fig.2 Velocity component contours with inner cylinder axial velocity

#### 4-Conclusion

In this study, the Taylor-Couette problem with the axial motion of the inner cylinder was investigated. The non-Newtonian fluid shear thinning fluid is considered and the Carreau-Bird model is used to model the viscosity dependence on the shear rate. Given that the governing equations in this problem have nonlinear high-order terms, irregular studies in these systems are important because of the existence of these nonlinear terms. Therefore, the Taylor number, which is the main criterion of the current regime, is used as the parameter of the dynamic steady state changes of the system.

The results that with increasing Shear Tinning effect the critical Taylor number increases and Taylor vortex are possible Are formed. Also with increasing axial Reynolds number, the Reynolds eddy velocities are accelerated and the critical Taylor number decreases. With increasing Reynolds axial number, the maximum viscosity area extends from the center of the two cylindrical space to the inner

cylinder wall. Taylor vortices also form in the form of repetitive circles on the axis of the cylinder, but taking into account the axial velocity of the inner cylinder, these vortices become axial spirals.

The stability of the current is affected by the Taylor control parameter rather than by the non-Newtonian fluid concentration and also the study of the dynamic behavior of the Taylor Vortex flow shows that with increasing axial Reynolds, the stability and the chaos are intensified. In other words, as the Reynolds axial increases, the behavior of the system undergoes a noticeable change. Also, with the increase of the non-Newtonian index, the instability in the system is further intensified.

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