



Simulation of nanofluid natural convection with presence of a magnetic field in a tilted cavity using lattice Boltzmann method

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ABSTRACT: In this work, a double multi-relaxation-time lattice Boltzmann method is used to simulate the magneto-hydrodynamics natural convection of nanofluid in a two-dimensional tilted square cavity. The cavity is filled with TiO_2 -water nanofluid at the presence of a magnetic field with an inclination angle of ϕ respect to horizontal plane. The proposed numerical scheme solves the flow field and temperature field by using a MRT-D2Q9 and MRT-D2Q5 lattice model, respectively. The obtained results indicate that augmentation of the magnetic field weakens the rate of heat transfer in the cavity. Also, in $\phi = 90^\circ$, the produced flow is not able to cover the entire cavity and is divided into two vortex; and the vortexes tend to take a symmetrical shape by increasing the Hartman number. At $\phi = 90^\circ$ and $Ha = 30$ and 60 , the isotherm contours become mushroom-shaped. In addition, it was observed that at high Hartman numbers ($Ha=60$), Lorentz force overcomes the buoyancy force and enhancement of solid volume fraction will not affect the rate of heat transfer in the cavity significantly.

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1- Introduction

The problem of natural convection of electrically conducting fluids in the presence of magnetic field is one of the major interesting research subjects due to its widely engineering applications. The magneto-hydrodynamics (MHD) natural convection has been studied by many researchers using experimental, analytical and numerical methods. Okada and Ozoe [1] carried out the experiments using molten gallium with Prandtl number of 0.024 in a cubical cavity heated from one side wall and cooled from the opposite wall with all other walls insulated in the presence of different magnetic field direction, and found that the external magnetic field in the vertical direction was more effective than the magnetic field applied parallel to the heated vertical wall. Pirmohammadi and Ghassemi [2] numerically studied the effect of a magnetic field on natural convection heat transfer inside a tilted square enclosure. Their results have shown that the variations of Nusselt number with inclination angle depends on the Hartmann number.

The Lattice Boltzmann Method (LBM) is a powerful numerical technique based on kinetic theory for simulating fluid flows and modeling the physics in fluids and nanofluids. In comparison with the conventional CFD methods, the advantages of LBM include simple calculation procedure, simple and efficient implementation for parallel computation, easy and strong handling of complex geometries, and others. Mahmoudi et al. [3] have analyzed the effects of a magnetic field on nanofluid flow in a cavity with a linear boundary condition using the LBM. The results show that the heat transfer rate increases with an increase of the Rayleigh number

but it decreases with an increase of the Hartmann number. Also for $Ra \geq 10^4$ the heat transfer and fluid flow depend strongly upon the direction of magnetic field. In addition, according the Hartmann number, it has been observed that the magnetic field direction controls the effects of nanoparticles. With regards to the LBM and 2-MRT-LBM model advantages, the purpose of this study is to simulate the MHD natural convection of water- TiO_2 nanofluid in a tilted square cavity using 2-MRT-LBM method. The flow field and temperature field are modeled by using a MRT-D2Q9 and MRT-D2Q5 lattice models, respectively. The results of 2-MRT-LBM are validated with previous numerical investigations and effects of all parameters (Rayleigh number, volume fraction, Hartmann number and magnetic field angle) on flow field and temperature distribution are also studied.

2- Methodology

The geometry of problem is shown in Fig. 1. Thermophysical properties of water and nanoparticles are also presented in Table 1.

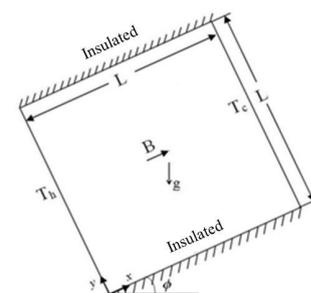


Figure 1. Geometry of problem

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Table 1. Thermophysical properties of water and nanoparticles

	Water	TiO ₂
<i>Pr</i>	6.2	-
ρ (kg/m ³)	997.1	4250
C_p (J/kgK)	4179	686.2
k (W/mK)	0.613	8.9538
β (K ⁻¹)	21×10 ⁵	0.9×10 ⁻⁵
μ (Ns/m ²)	0.001003	-

The following equation is used to solve the flow field [4]:

$$|f(\mathbf{x} + \mathbf{c}_i \Delta t, t + \Delta t)\rangle - |f(\mathbf{x}, t)\rangle = -M^{-1}S[|m(\mathbf{x}, t)\rangle - |m^{eq}(\mathbf{x}, t)\rangle] + |\mathbf{F}(\mathbf{x}, t)\rangle, \quad (1)$$

in which

$$\begin{aligned} \mathbf{F} &= F_x \mathbf{i} + F_y \mathbf{j}, \\ F_x &= 3\omega_i \rho_{nf} g \beta_{nf} \sin \phi (T - T_m), \\ F_y &= 3\omega_i \rho_{nf} ((g \beta_{nf} \cos \phi (T - T_m) - Av) \end{aligned} \quad (2)$$

$$A = \frac{Ha^2 \nu}{n^2}$$

The mapping between discrete velocity space and moment space is achieved by the transformation matrix *M* which maps the vector $|f(\mathbf{x}, t)\rangle$ to the vector $|m(\mathbf{x}, t)\rangle$.

$$|m\rangle = M |f\rangle, |f\rangle = M^{-1} |m\rangle. \quad (3)$$

$$M = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -4 & -1 & -1 & -1 & -1 & 2 & 2 & 2 & 2 \\ 4 & -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 & 1 & -1 & -1 & 1 \\ 0 & -2 & 0 & 2 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 & 1 & -1 & -1 \\ 0 & 0 & -2 & 0 & 2 & 1 & 1 & -1 & -1 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \end{bmatrix}, \quad (4)$$

The following equation is used to solve the temperature field [4]:

$$|g(\mathbf{x} + \mathbf{c}_i \Delta t)\rangle - |g(\mathbf{x}, t)\rangle = -\Omega[|g(\mathbf{x}, t)\rangle - |g^{eq}(\mathbf{x}, t)\rangle], \quad (5)$$

Ω is the collision operator and $\Omega = M^{-1}SM$ and *M* is:

$$M = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ -4 & 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & 1 & -1 \end{bmatrix}, \quad (6)$$

Finally, the incompressible Navier-Stokes equations can be derived from this method using a Chapman-Enskog multi-scale expansion.

The viscosity of the nanofluid is given by Brinkman model [5] as:

$$\mu_{nf} = \frac{\mu_f}{1 - \phi^{2.5}} \quad (7)$$

The effective thermal conductivity of the nanofluid can be approximated by the Maxwell-Garnetts model as [5]:

$$k_{nf} = k_f \left[\frac{k_s + 2k_f + 2\phi(k_f - k_s)}{k_s + 2k_f - \phi(k_f - k_s)} \right] \quad (8)$$

3- Results and Discussion

Fig. 2 shows the effect of variations of inclination angle on streamlines for various Hartmann numbers and $Ra=10^5$ and $\phi = 3\%$. As can be shown in this figure at $Ha=30$ and 60 , streamlines form circular shape with increasing the inclination angle to 60° which is due to the fact that the horizontal component of buoyancy force is being stronger. At $Ha=60$, the Lorentz force is stronger and is applied on the opposite direction to velocity vertical components and therefore, the streamlines form vertical shape like as rectangular. At $\phi=90^\circ$, the created flow is not able to encompass the entire cavity and is divided to two vortexes.

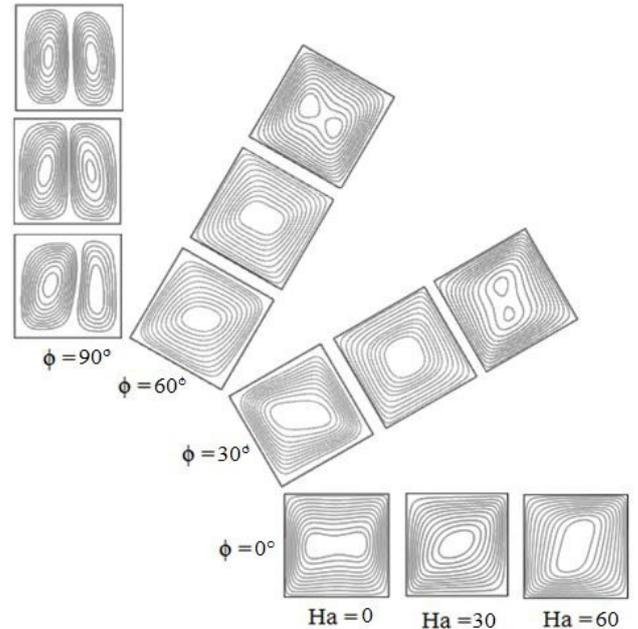


Figure 2. Streamlines at various inclination angles, $Ra=10^5$, $\phi=3\%$ and $Ha=0, 30$, and 60 .

Fig. 3 shows the variations of average Nusselt number with solid volume fraction for various Hartman numbers and inclination angles and $Ra=10^5$. It can be seen that the average Nusselt number increased with increasing the inclination angle from 0° to 30° and decreased with decreasing the inclination angle from 30° to 90° .

4- Conclusions

- The results of this study have been compared with previous researches and it has been shown that there is a good agreement between the results of 2-MRT-LBM model with previous studies.

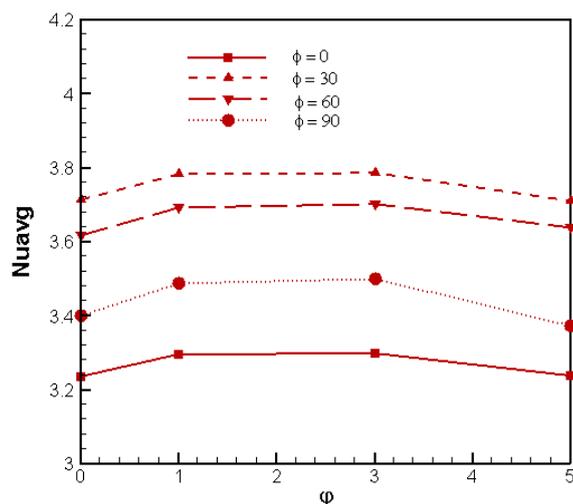


Figure 3. Average Nusselt number for different ϕ and inclination angle at $Ha=30$ and $Ra=10^5$.

- At $Ra=10^5$ and $Ha=0$ the streamlines form circular shape with increasing ϕ from 0° to 60° .
- At $Ra=10^5$ and $Ha=30, 60$ and $\phi=3\%$ and $\phi=90^\circ$ the created flow is not able to encompass the entire cavity and is divided to two vortices.
- At $Ha=30$ and $Ra=10^5$ and with increasing the inclination angle from 0° to 30° , the average Nusselt number increased

and with increasing of inclination angle from 30° to 90° it will decrease.

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