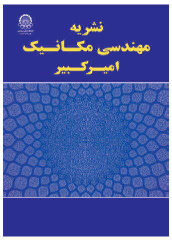

Amirkabir Journal of Mechanical Engineering

Amirkabir J. Mech. Eng., 50(5) (2018) 353-356
DOI: 10.22060/mej.2017.11937.5223

Nonlinear Longitudinal Free Vibration of a Rod Undergoing Finite Strain

B. Soleimani Roody, A. Fotuhi* , M. Jalili

Department of Mechanical Engineering, Yazd University, Yazd, Iran.

ABSTRACT: Rods are one of significant engineering's structures and vibration analysis of a rod because of extended application of it in engineering is very important. Therefore, understanding of longitudinal nonlinear vibration of rod with different boundary conditions and large amplitude is very useful. In this paper, vibration of a rod with different boundary conditions undergoing finite strain, without simplification in strain-displacement relations, is investigated. For obtaining governing equation, Green-Lagrange strain, structural damping and Hamilton principle are used and then Galerkin method is employed to convert nonlinear partial differential equation to nonlinear ordinary differential equation. In spite of many papers that only use of cubic term for nonlinearity, the governing equation has quadratic and cubic terms. The equations with and without damping, are solved with multiple time scales method. In order to verify the accuracy of this method, the results are compared with results of Runge-Kutta numerical method, which have good accuracy. Finally sensitivity analysis for understanding of influence of nonlinear coefficients on rod vibration answer is done.

Review History:

Received: 2 September 2016
Revised: 4 November 2016
Accepted: 25 December 2016
Available Online: 1 January 2017

Keywords:

Nonlinear vibration
Finite strain
Green-Lagrange strain
Multiple scales method
Sensitivity analysis

1- Introduction

The significance of vibration of rod could be found because of its application in wide range of mechanical systems such as aircraft wings, robot arms and the design of towers and tall buildings [1]. Accordingly, many articles are published on the vibration analysis of different kind of rods. But many studies for modeling geometrical nonlinearity that considers nonlinear terms in strain-displacement equation, has used the large deformation and small strain with Eulerian description and has been performed on basis of Von-Kármán assumptions, that extraction of Von-Kármán equation was explained by Timoshenko [2].

In this research, vibration of a rod affected by finite strain (large deformation and finite strain) which is subjected to different boundary conditions is investigated. Therefore, all nonlinear terms of Green-Lagrange strain tensor are considered according to finite strain assumption. Analytical expressions for time responses with and without damping are derived. In spite of many papers that only use of cubic term for nonlinearity, the governing equation has quadratic and cubic terms. In order to verify the accuracy of this method, the results are compared with results of Runge-Kutta numerical method, which have good accuracy. Finally, sensitivity analysis for understanding of influence of nonlinear coefficients on rod vibration answer is done.

2- Equation of Motion

Lagrangian strain for rod is

$$E_{xx} = \left(1 + \frac{1}{2}\frac{\partial U}{\partial X}\right)\frac{\partial U}{\partial X} \tag{1}$$

Corresponding author, E-mail: afotuhi@yazd.ac.ir

Then, with considering elastic material, Piola-Kirchhoff stress in reference configuration is obtain

$$P_{xx} = YE_{xx} = Y\left[\frac{\partial U}{\partial X} + \frac{1}{2}\left(\frac{\partial U}{\partial X}\right)^2\right] \tag{2}$$

where Y is elasticity module.

By using of Hamilton principle, the equations of motion are obtained as follow with and without damping, respectively:

$$\frac{\partial^2 U}{\partial X^2}\left[1 + 3\frac{\partial U}{\partial X} + \frac{3}{2}\left(\frac{\partial U}{\partial X}\right)^2\right] + \frac{\mu}{Y}\frac{\partial}{\partial t}\frac{\partial^2 U}{\partial X^2}\left[1 + 3\frac{\partial U}{\partial X} + \frac{3}{2}\left(\frac{\partial U}{\partial X}\right)^2\right] = \frac{\rho}{Y}\frac{\partial^2 U}{\partial t^2} \tag{3}$$

$$\frac{\partial^2 U}{\partial X^2}\left[1 + 3\frac{\partial U}{\partial X} + \frac{3}{2}\left(\frac{\partial U}{\partial X}\right)^2\right] = \frac{\rho}{Y}\frac{\partial^2 U}{\partial t^2} \tag{4}$$

By assuming the vibration of the rod as single-harmonic, its displacement can be written as

$$u(x,\tau) = W(x)\theta(\tau) \tag{5}$$

where $W(x)$ must satisfy the boundary conditions, $W(x)$ is represented for different boundary conditions in Table 1.

Table 1. Value of $W(x)$ for different boundary conditions [3]

boundary conditions	$W(x)$
fixed-free	$\sin\left(\frac{\pi x}{2}\right)$
fixed-fixed	$\sin(\pi x)$
fixed-mass	$\sin(\beta x)$
free-spring	$\cos(\gamma x)$

For fixed-mass rod β is solution of equation $\beta \tan\beta = \rho AL/M$ and for free-spring rod γ is answer of equation $\gamma \cot\gamma = AY/LK$. Using the Galerkin method, the equation according to time variable, for free longitudinal vibration of a rod with and without nonlinear damping are obtained as

$$\ddot{\theta} + \mu_1\left(\alpha_3\theta^2 + \alpha_2\theta + \alpha_1\right)\dot{\theta} + \alpha_3\theta^3 + \alpha_2\theta^2 + \alpha_1\theta = 0 \quad (6)$$

$$\ddot{\theta} + \alpha_3\theta^3 + \alpha_2\theta^2 + \alpha_1\theta = 0 \quad (7)$$

In order to solve the nonlinear free vibration, Eqs.(6) and (7) solved with multiple scale method. After some manipulations, the expression of the for a rod with and without nonlinear damping are obtained as

$$\begin{aligned}\theta &= \eta\cos\left(\omega_0 t + \xi\right) + \varepsilon\left[\frac{\alpha_2}{6\omega_0^2}\eta^2\cos\left(2\omega_0 t + 2\xi\right)\right] \\ &-\varepsilon\left[\frac{\mu_1\alpha_2}{6\omega_0}\eta^2\sin\left(2\omega_0 t + 2\xi\right)\right] + \varepsilon\left[\frac{\alpha_3}{32\omega_0^2}\eta^3\cos\left(3\omega_0 t + 3\xi\right)\right] \\ &-\varepsilon\left[\frac{\mu_1\alpha_3}{32\omega_0}\eta^3\sin\left(3\omega_0 t + 3\xi\right)\right] - \varepsilon\left[\frac{\alpha_2}{2\omega_0^2}\eta^2\right]\end{aligned} \quad (8)$$

$$\begin{aligned}\theta &= \varepsilon\eta\cos\left(\omega t + \xi_0\right) - \frac{\varepsilon^2\eta^2\alpha_2}{2\alpha_1}\left[1 - \frac{1}{3}\cos\left(2\omega t + 2\xi_0\right)\right] \\ &+\left[\frac{3\alpha_3\alpha_1 + 2\alpha_2}{24\alpha_1^2}\right]\varepsilon^3\eta^3\cos\left(3\omega t + 3\xi_0\right) + O\left(\varepsilon^4\right)\end{aligned} \quad (9)$$

Dimensionless vibration amplitude for fixed-fixed rod and damping coefficient 0.02 are shown in Fig.1. In order to verify the accuracy of this method, the results are compared with results of Runge-Kutta numerical method, which have good accuracy.

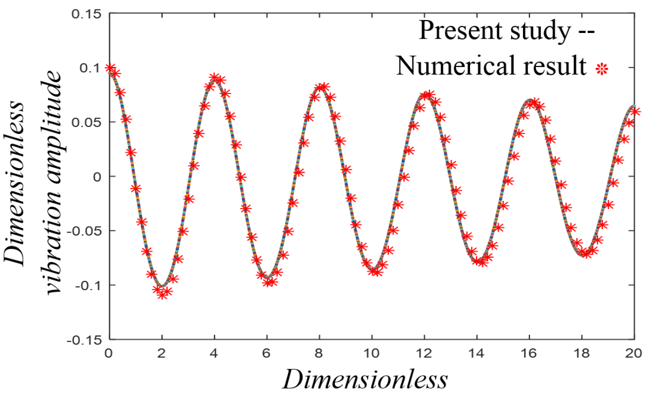

Fig.1. Dimensionless vibration amplitude for fixed-fixed rod

The ratio of nonlinear to linear frequency of fixed-mass rod for different ratios of *ρAL/M* and for free-spring rod for different ratios of *AY/LK* is shown in Figs.2 and 3. Changing of nonlinear to linear frequency versus dimensionless initial amplitude for rods with other boundary conditions has same behavior that determines the importance of nonlinear analysis in large initial conditions. Also, figures show that for fixed-mass rod with increase of *ρAL/M* and for free-spring rod with decrease of *AY/LK*, difference between nonlinear to linear frequency ratio increases.

3- Sensitivity Analysis

Finally, for understanding of influence of nonlinear coefficients on rod vibration answer, sensitivity analysis with calculation of J function is done.

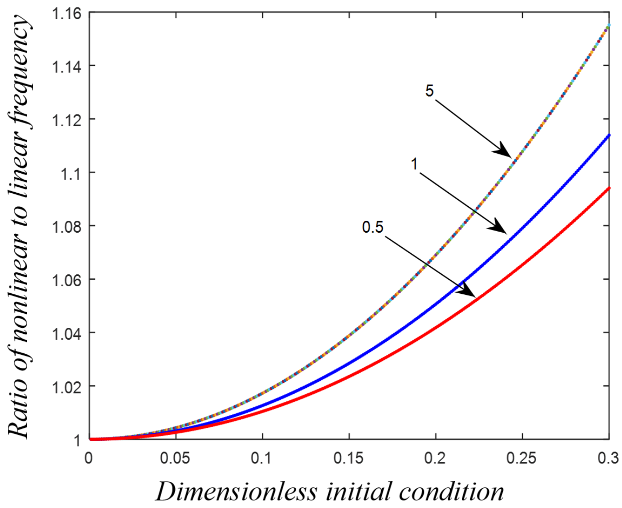

Fig.2. Influence of initial amplitude on nonlinear to linear frequency ratio

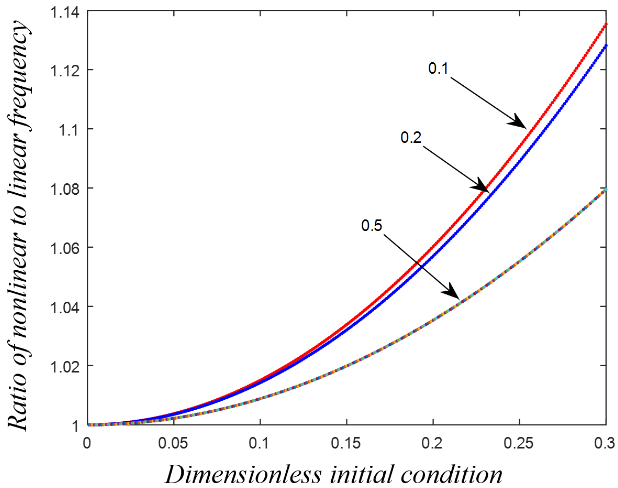

Fig.3. Influence of initial amplitude on nonlinear to linear frequency ratio

Table 2. Amount of J function

boundary conditions	$\alpha_2(-10\%)$	$\alpha_2(-10\%)$	$\alpha_2(-10\%)$	$\alpha_2(-10\%)$
fixed-free	0.000337	0.000337	0.1271	0.1111
fixed-fixed	0.00140	0.00140	0	0
fixed-mass	0.000347	0.000347	0.1214	0.1080
free-spring	0.000151	0.000151	0.2361	0.2437

4- Conclusions

Some of the important achievements of this research are listed as following:

- With increasing of initial amplitude, difference of nonlinear to linear frequency increases that determines the importance of nonlinear analysis in large initial conditions.

- Nonlinear analysis of fixed mass rod with smaller mass and nonlinear analysis of free-spring rod with harder spring is more important.
- With considering complete Green-Lagrangian strain, term of appeared in governing equations and sensitivity analysis showed that changing of is very important effect on vibration response of rod. Therefore, considering complete Green-Lagrangian strain is necessary for achieving exact vibration response of rod.

References

[1] Raj A., Sujith R., 2005, “Closed-form solutions for the free longitudinal vibration of inhomogeneous rods”. Journal of sound and vibration, 283(3), 1015-1030

[2] Timoshenko S., Woinowsky-Krieger S., 1959, “Theory of plates and shells”, Vol. 2, McGraw-hill New York

[3] Rao S S. “Vibration of continuous systems”, 2007, John Wiley & Sons.

Please cite this article using:

B. Soleimani Roody, A. Fotuhi , M. Jalili, Nonlinear Longitudinal Free Vibration of a Rod Undergoing Finite Strain, *Amirkabir J. Mech. Eng.*, 50(5) (2018) 353-356.

DOI: 10.22060/mej.2017.11937.5223